Rooting for Efficiency:
Mechanised Reasoning about Array-Based Trees in Separation Logic

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For CPP 2024

Pointer-Based Trees

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// binary tree
struct node {
  struct node
    *parent,
    *left_child,
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};
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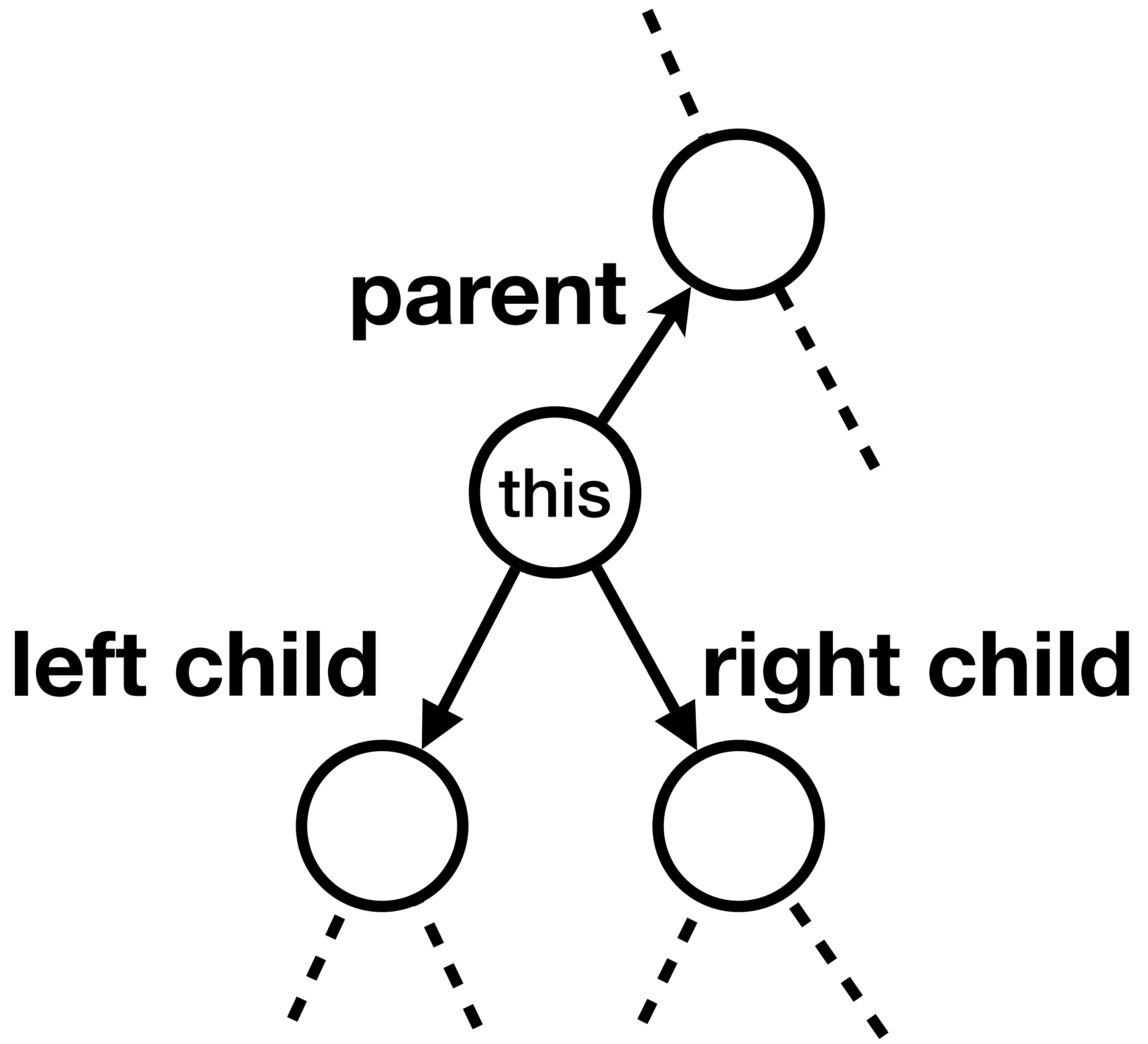

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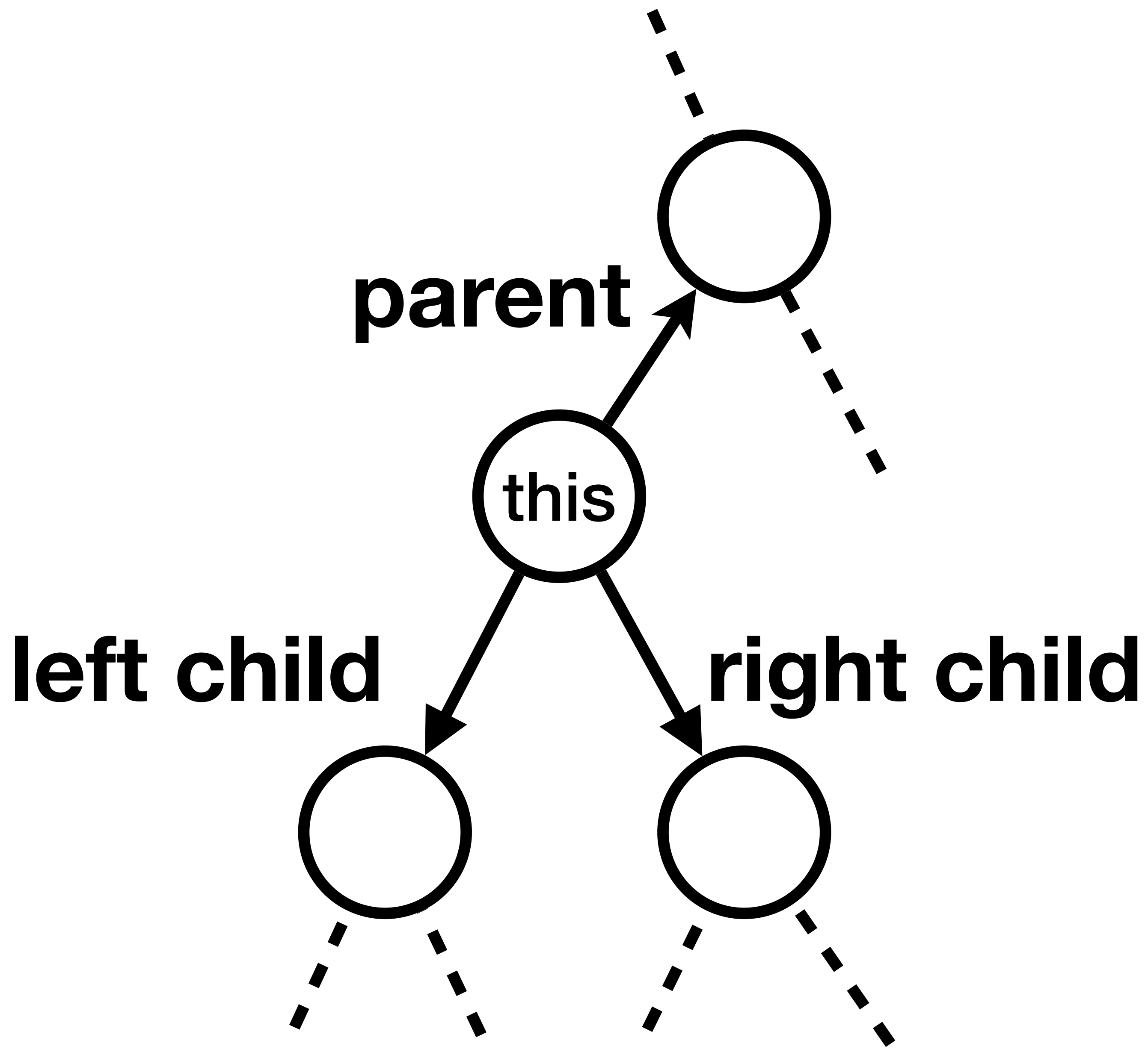

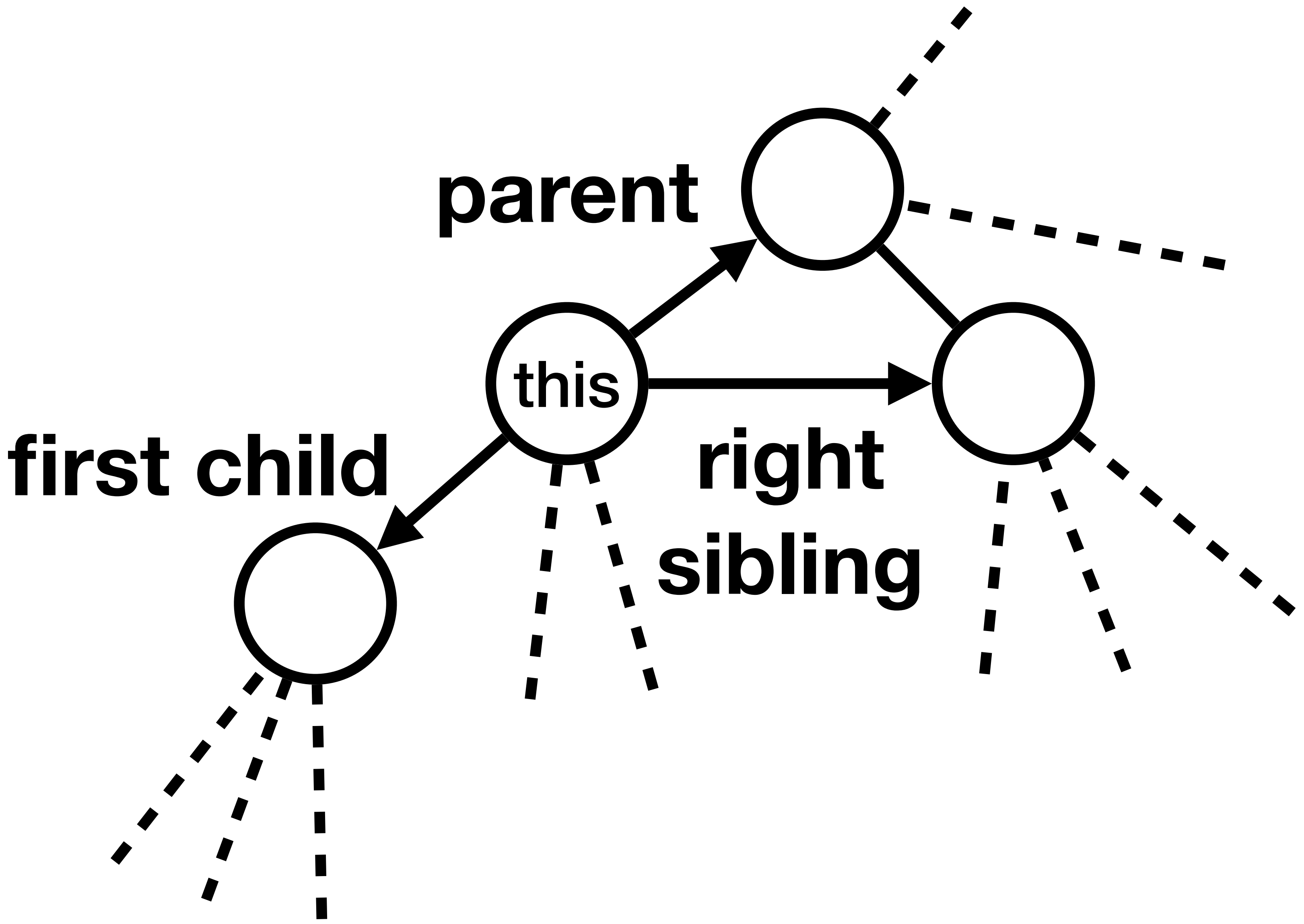

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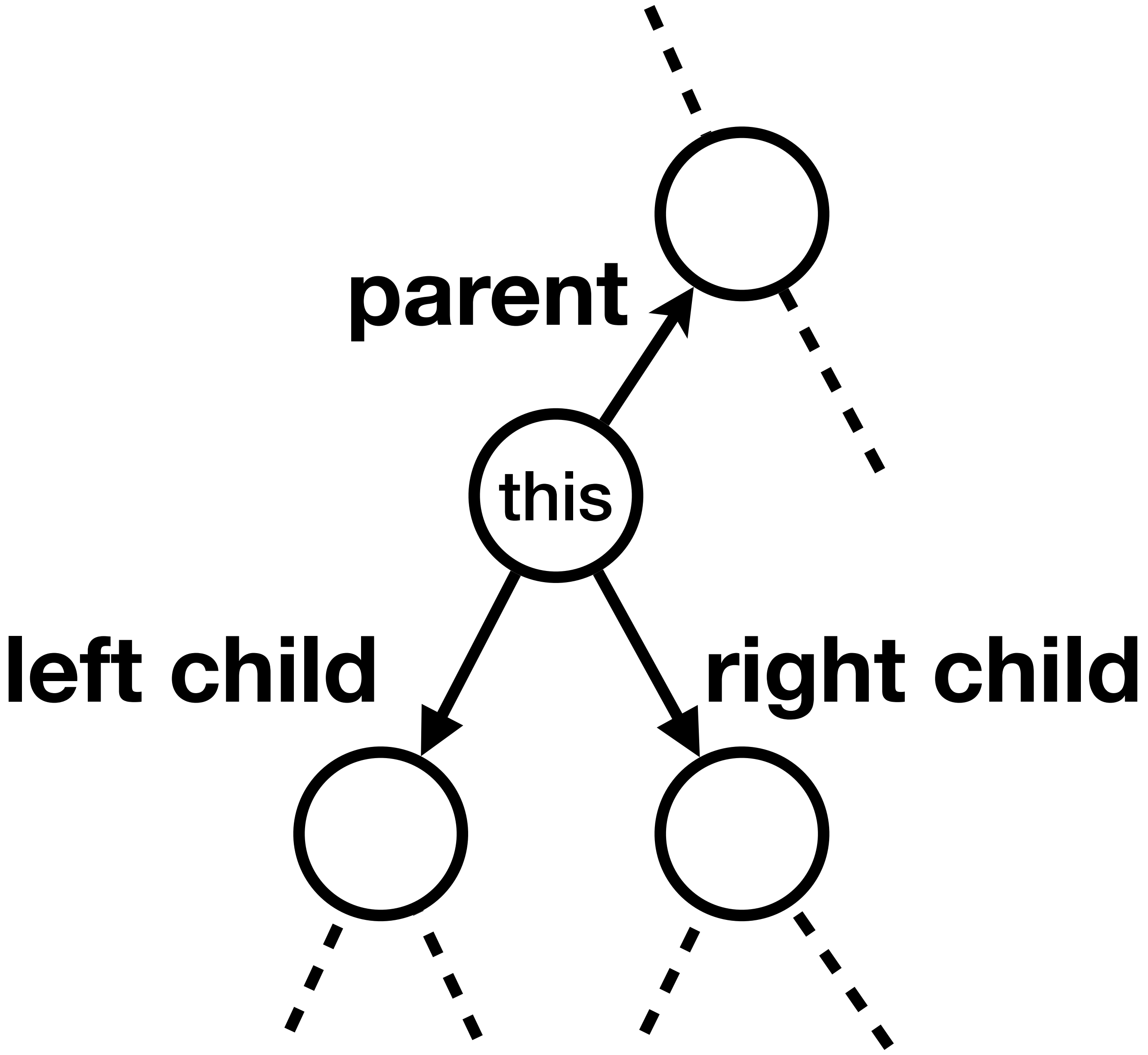

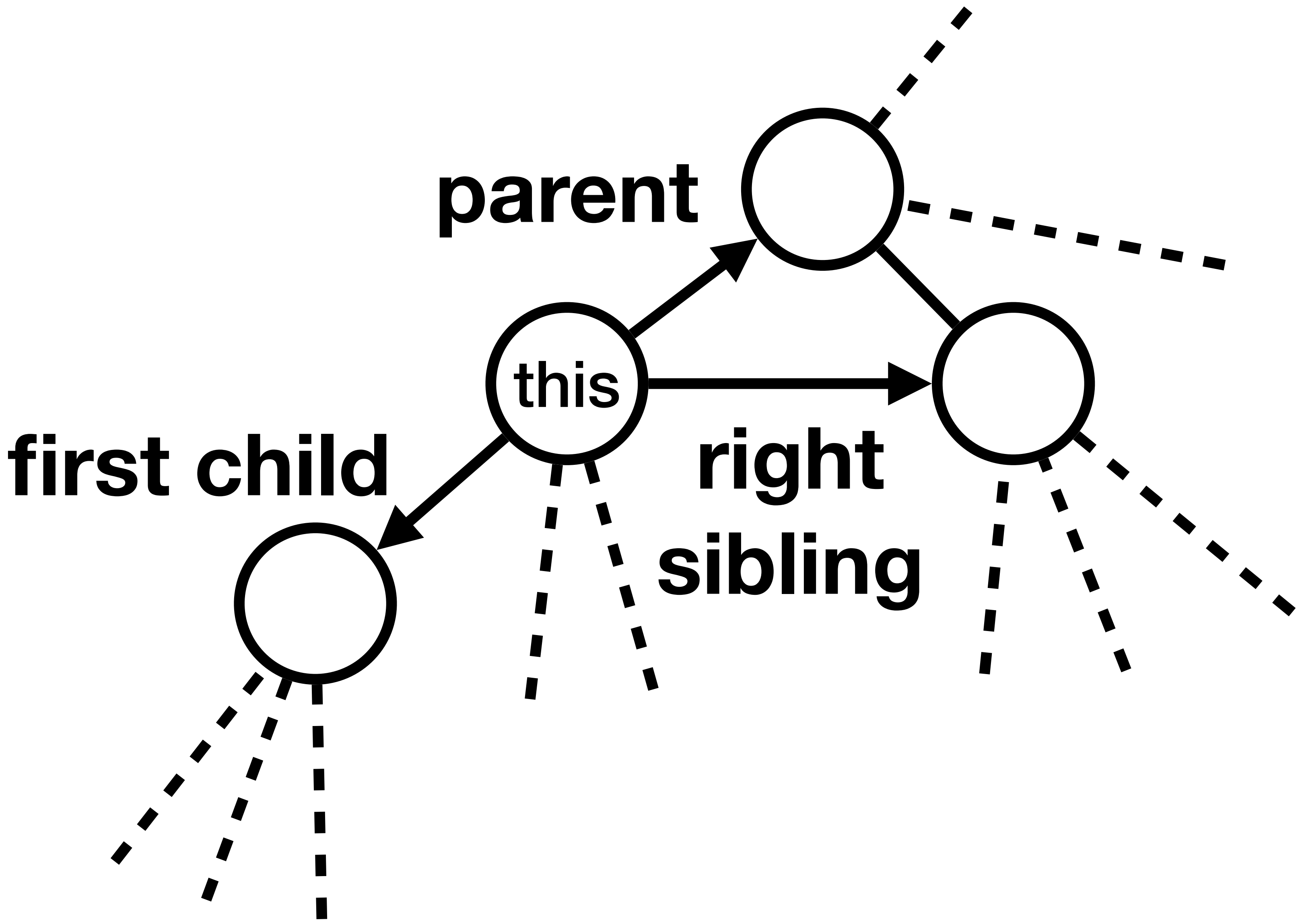

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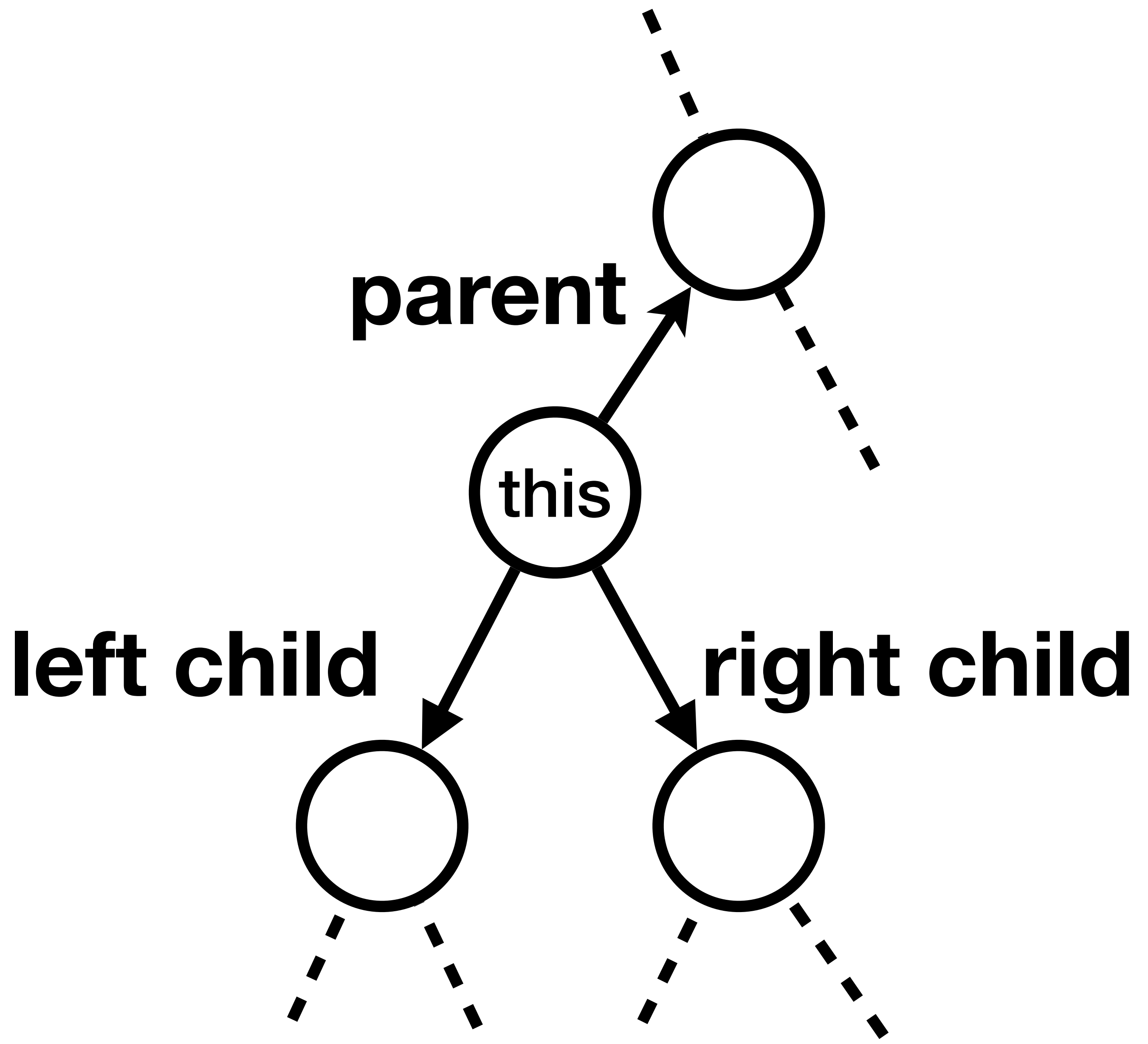

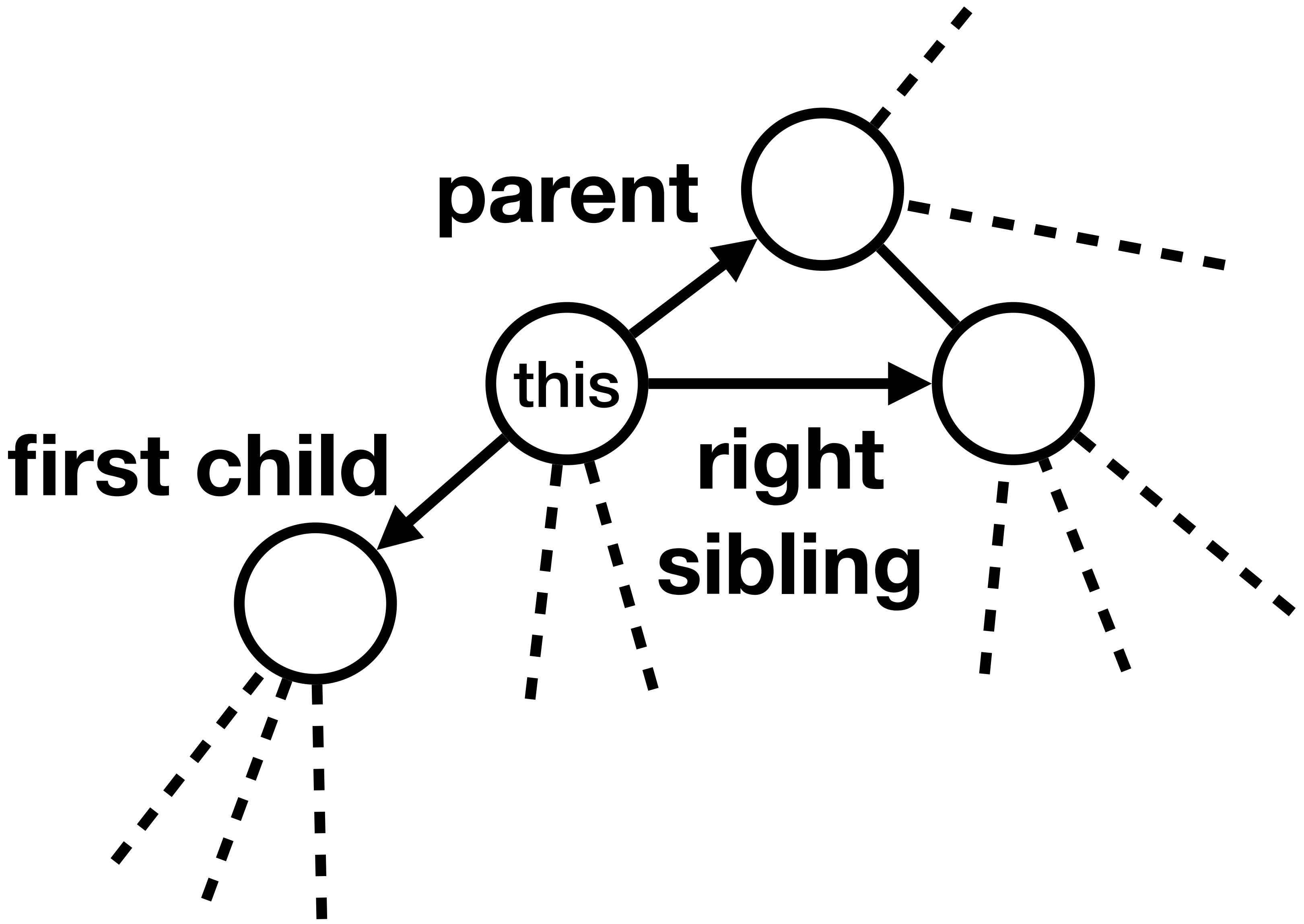

Array-Based Trees

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const int NIL = -1;
struct node tree[N];
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- Use a `struct` array to store a tree

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Array-Based Trees

- Use a `struct` array to store a tree
- Replace pointers with array indices
- Use a dedicated integer (e.g., `-1`) to represent `NULL` pointer

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Array-Based Trees (Cont’d)

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  - Potential time/space efficiency benefits
    - E.g., random access to a node's information
- **But formally reasoning about array-based trees can be challenging!**

Array-Based Trees in Separation Logic

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$$\text{tree\_rep}_{\text{arr}}(p, tr) \triangleq \exists \ell, \quad \lceil \text{tree\_proj}(tr, \ell) \rceil$$
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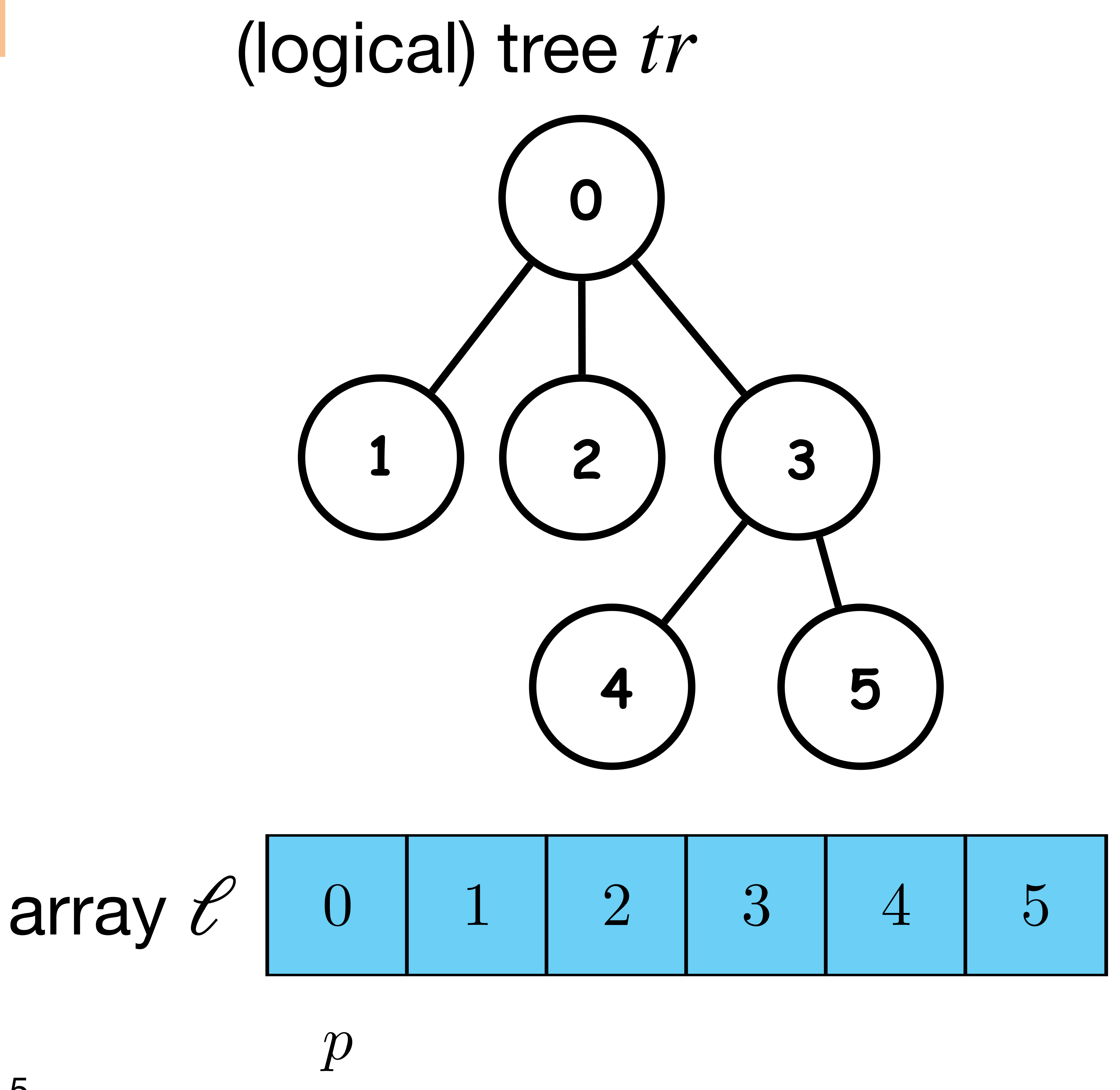

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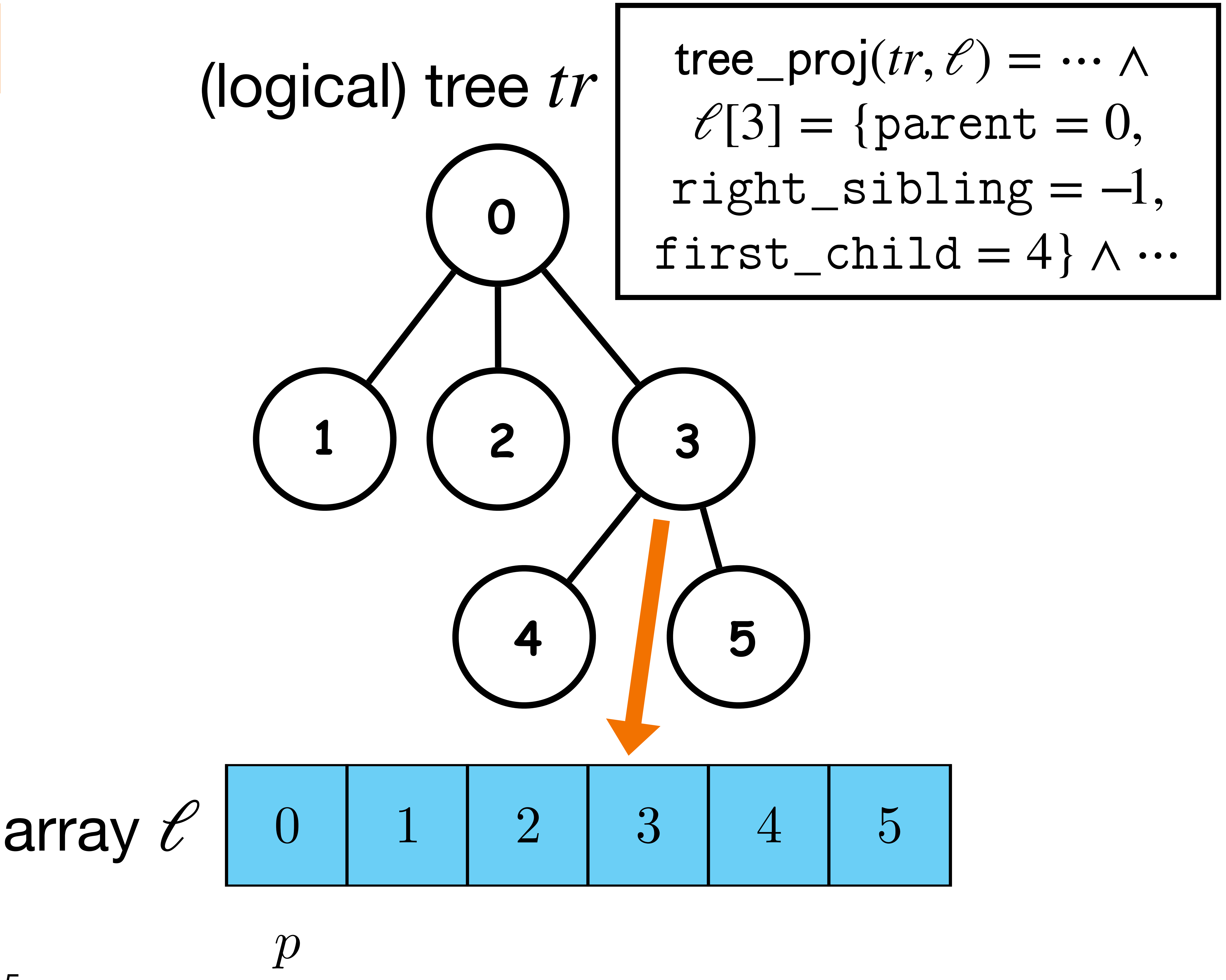

Challenge #1: Structure Changing Operation

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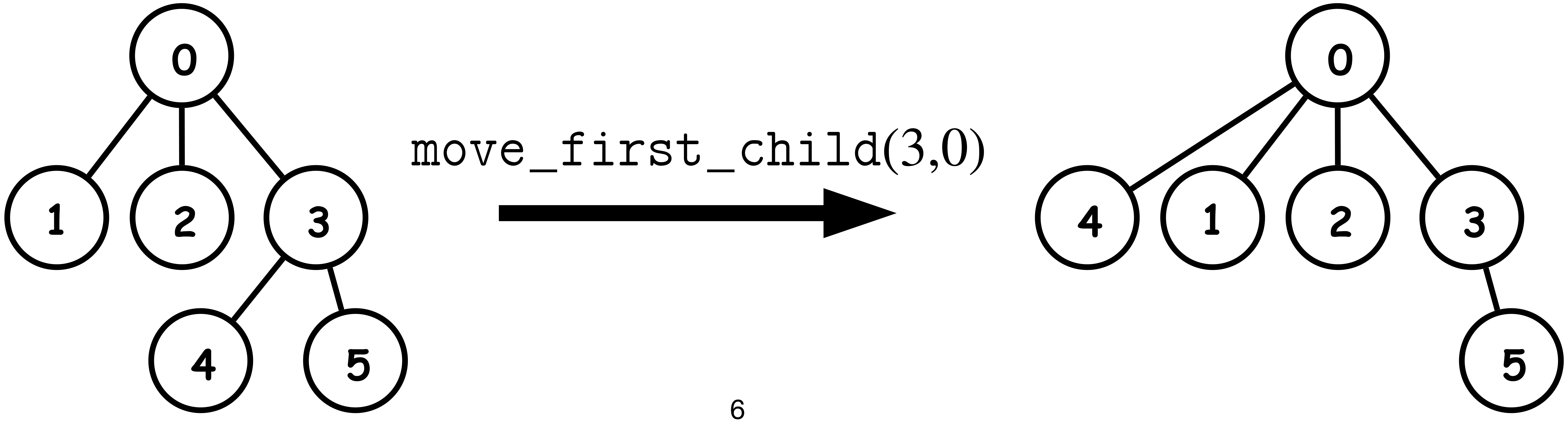

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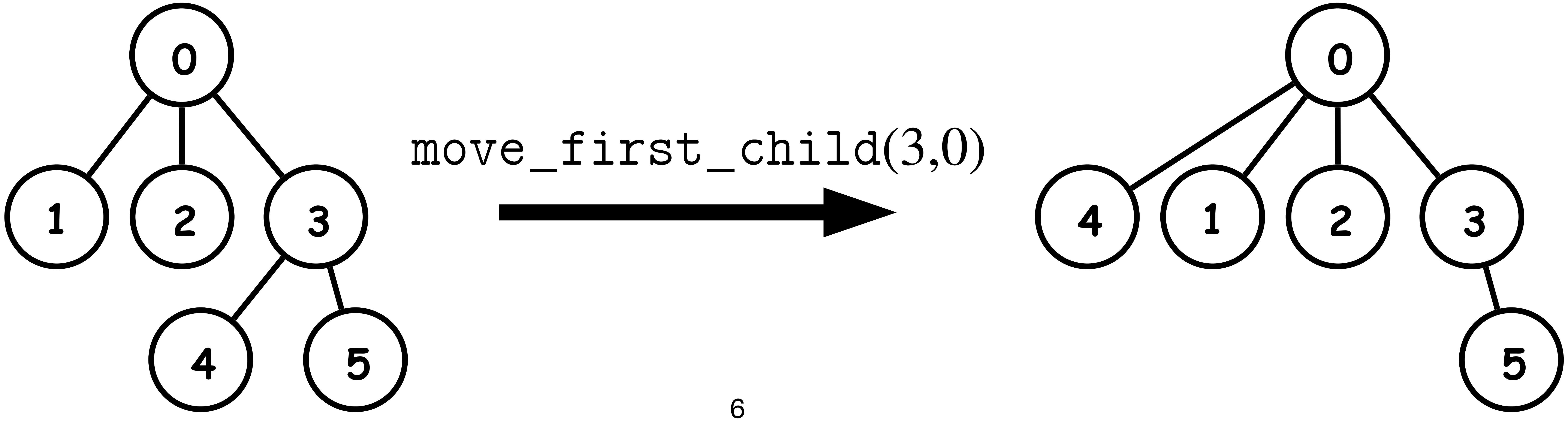

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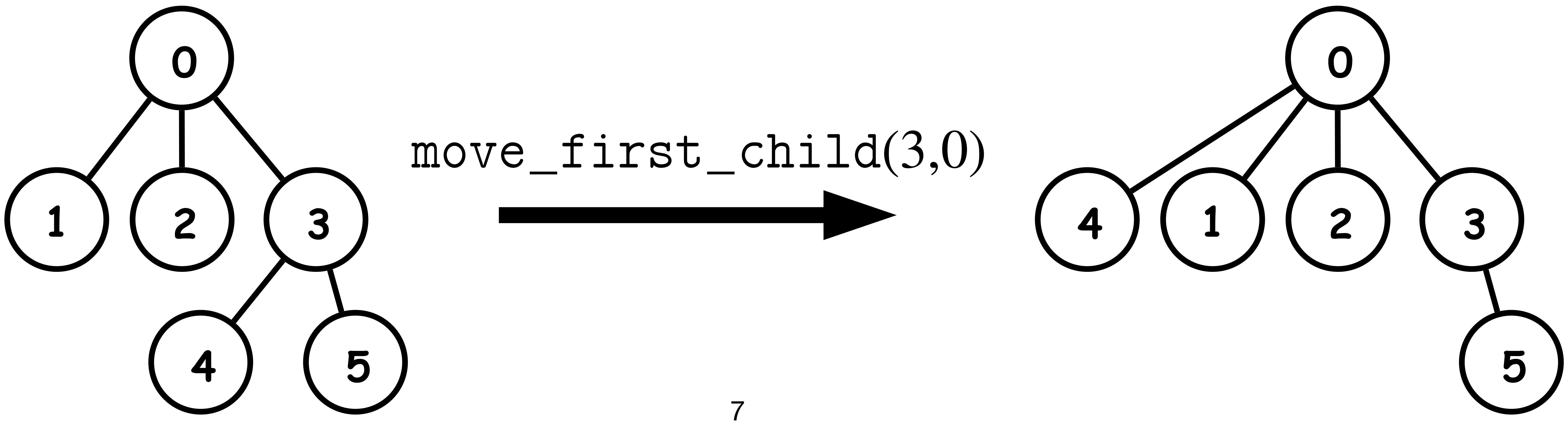

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- Need to specify what parts of the array have changed, and how they change

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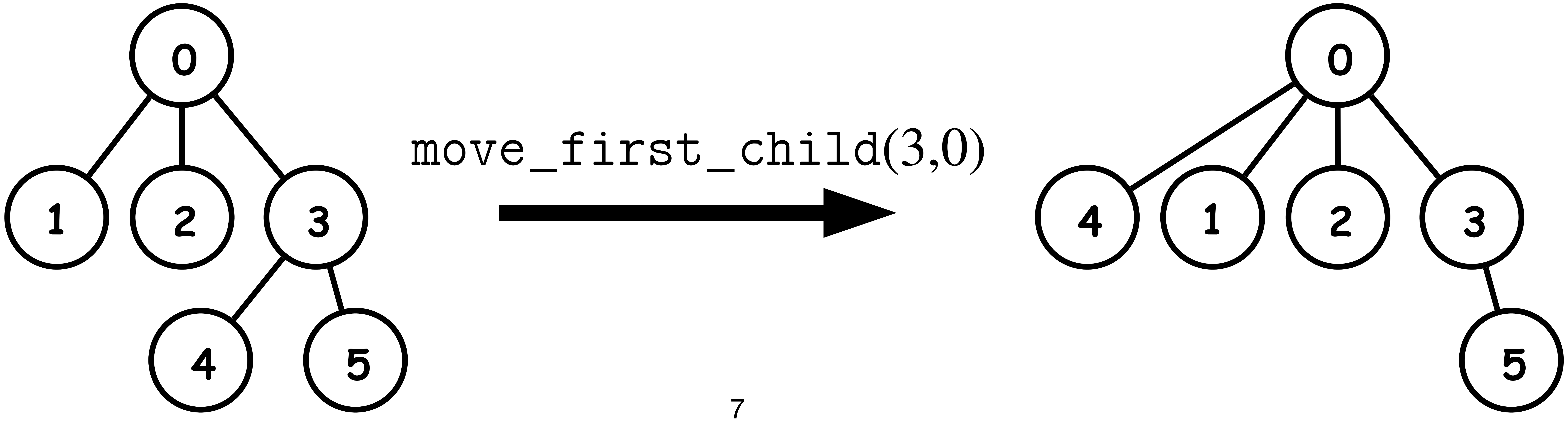

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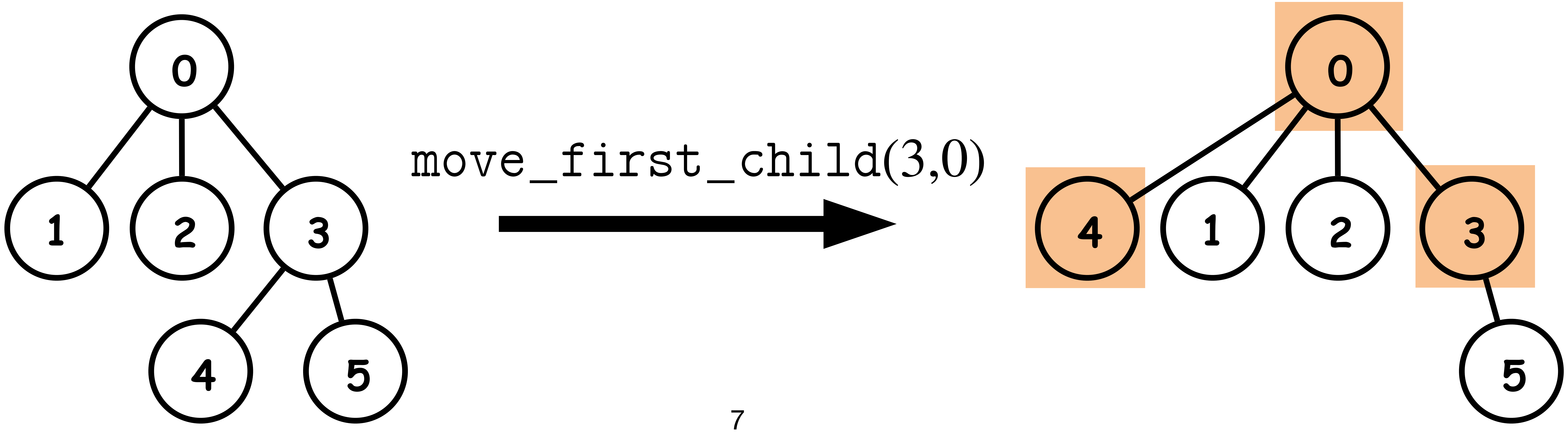

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- Need to specify what parts of the array have changed, and how they change
- Need to prove that the other parts are kept intact

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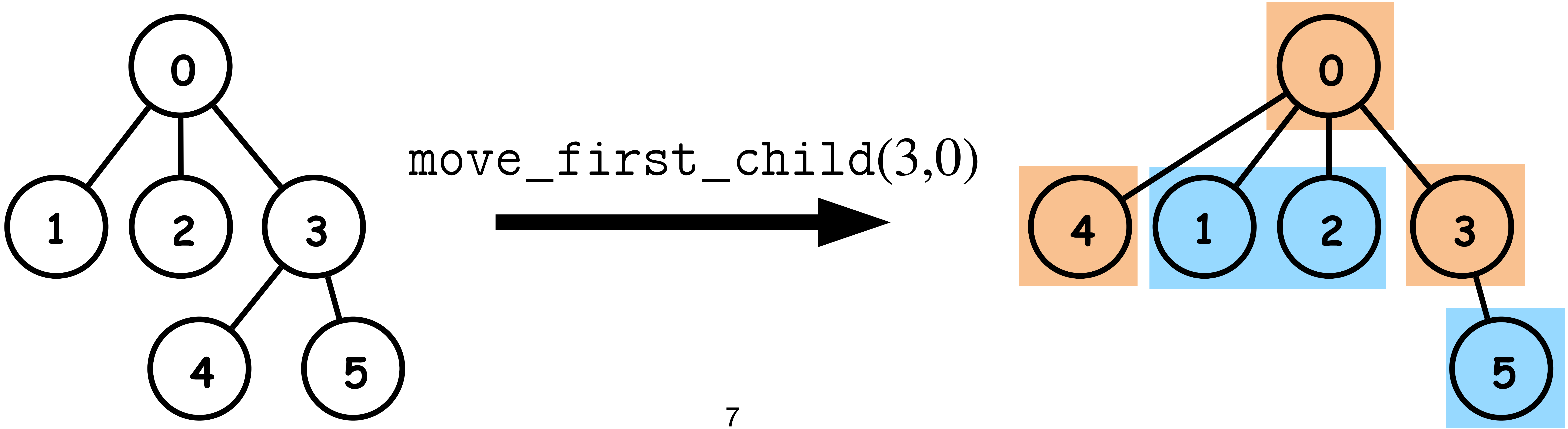

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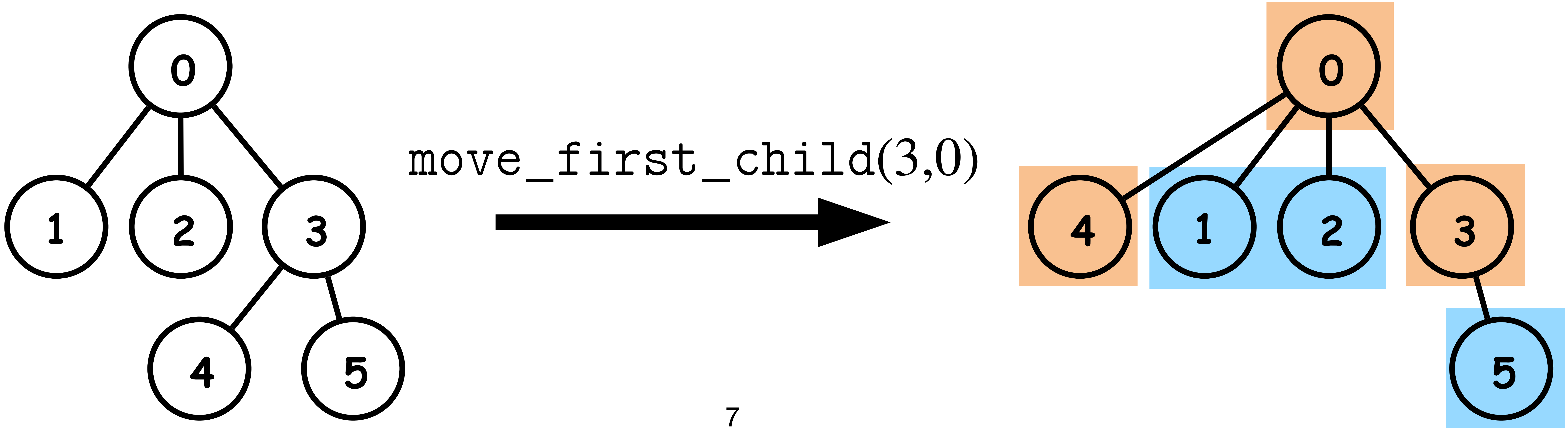

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- Need to prove that the other parts are kept intact
- $\Longrightarrow$ Want a “frame rule” to do **localised** reasoning on changed parts only
- $\Longrightarrow$ Need to “separate” the array, but a separated part cannot be represented by $\mathsf{tree\_rep}_{\mathsf{arr}}$

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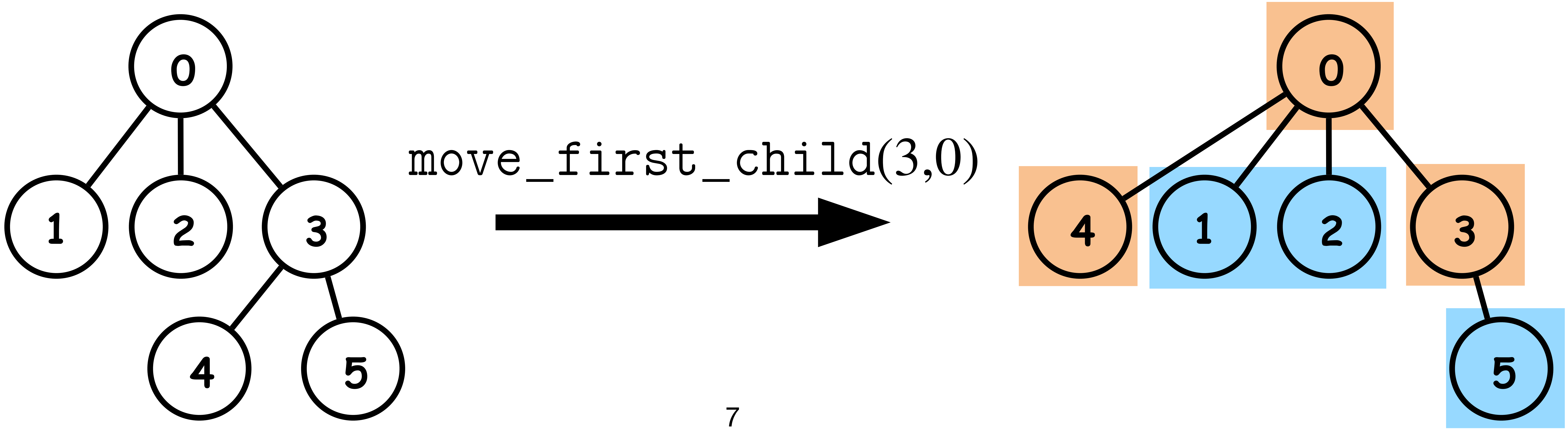

Challenge #2: Non-Recursive Traversal

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void nonrec_traversal (int root) {
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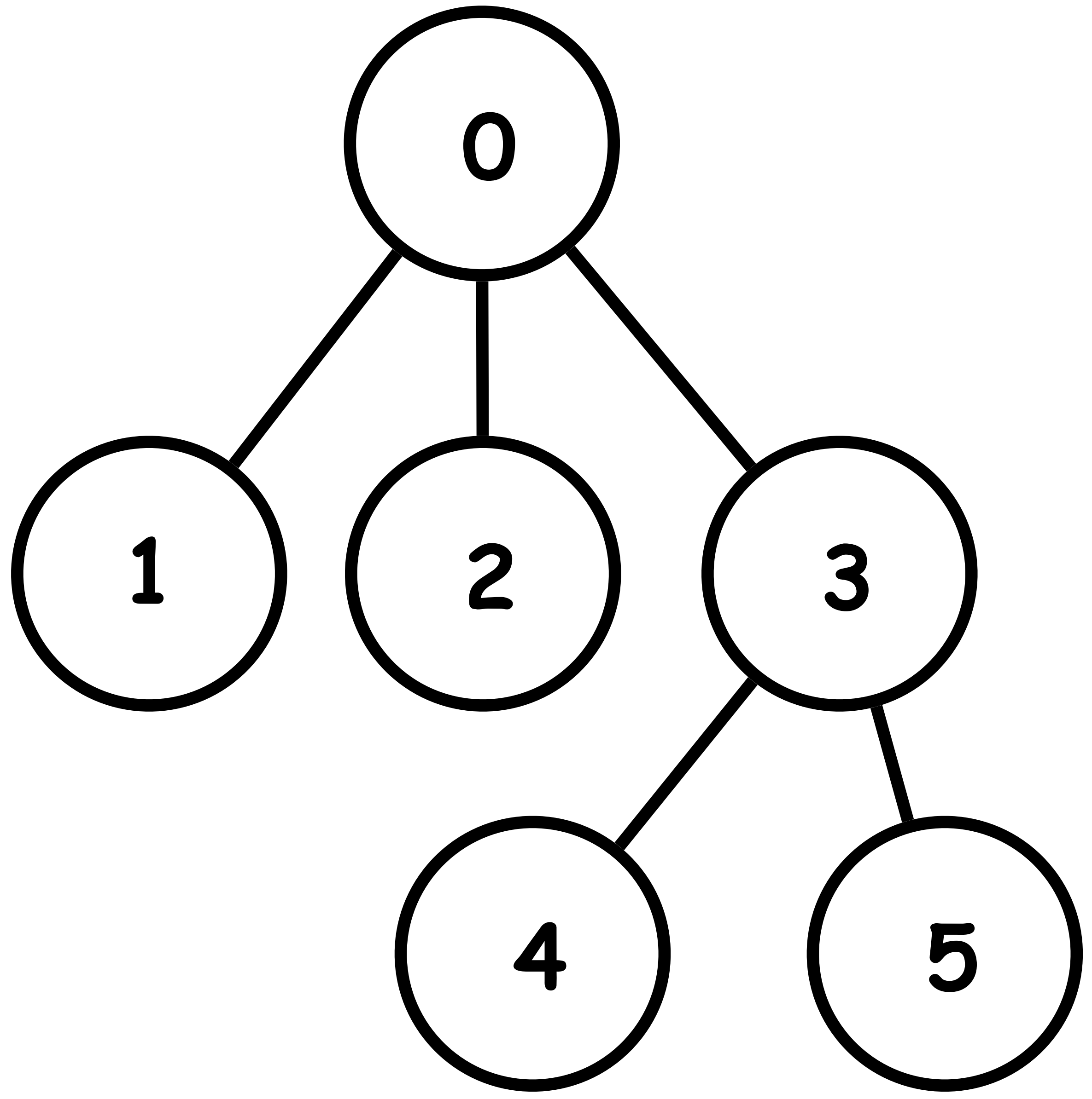

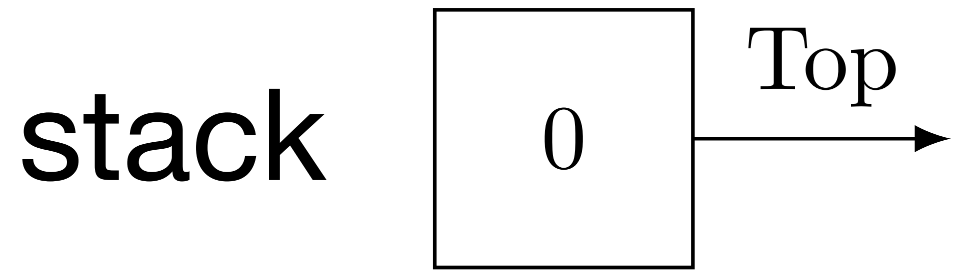

node visiting order

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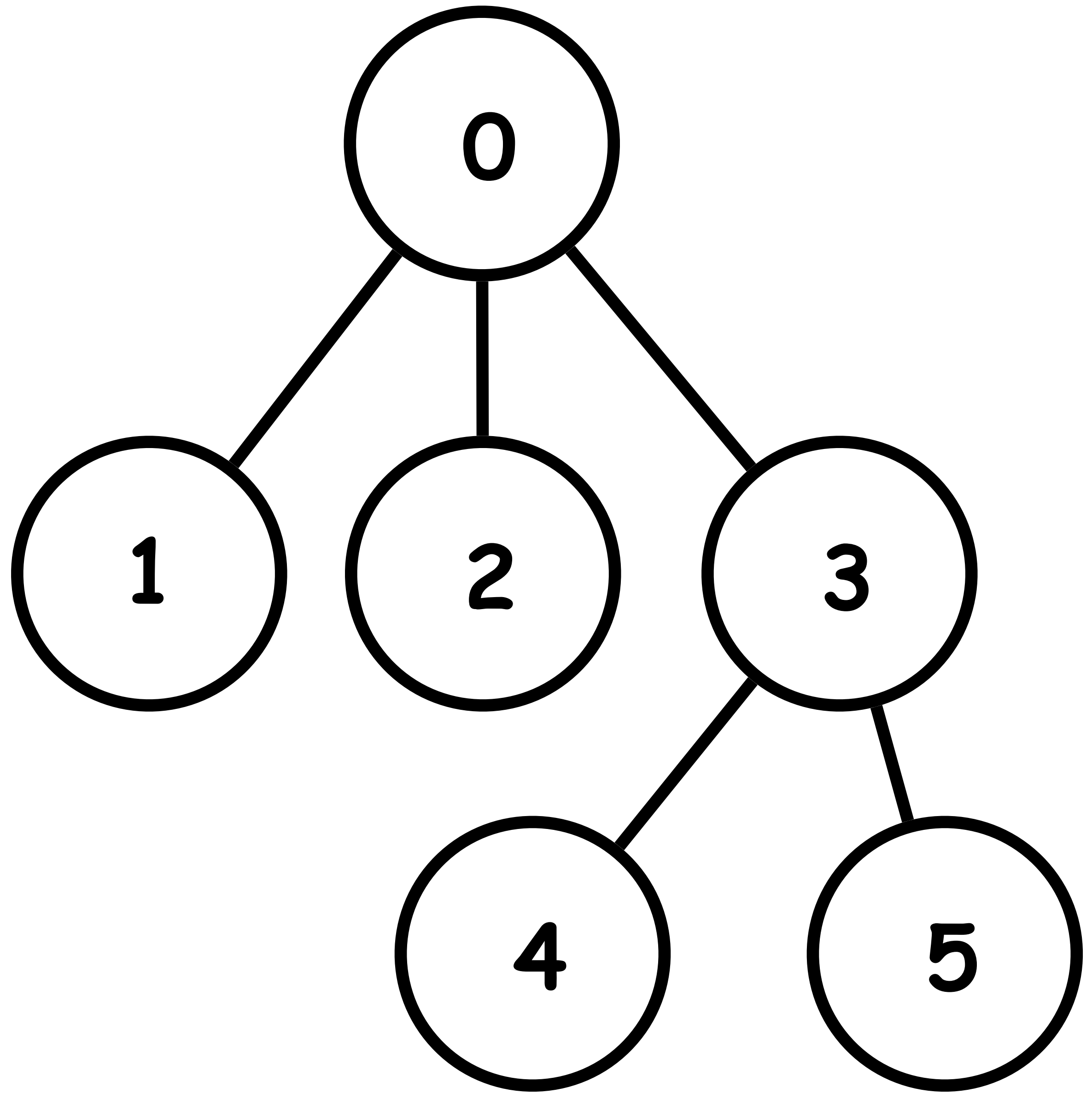

stack

node visiting order | 0

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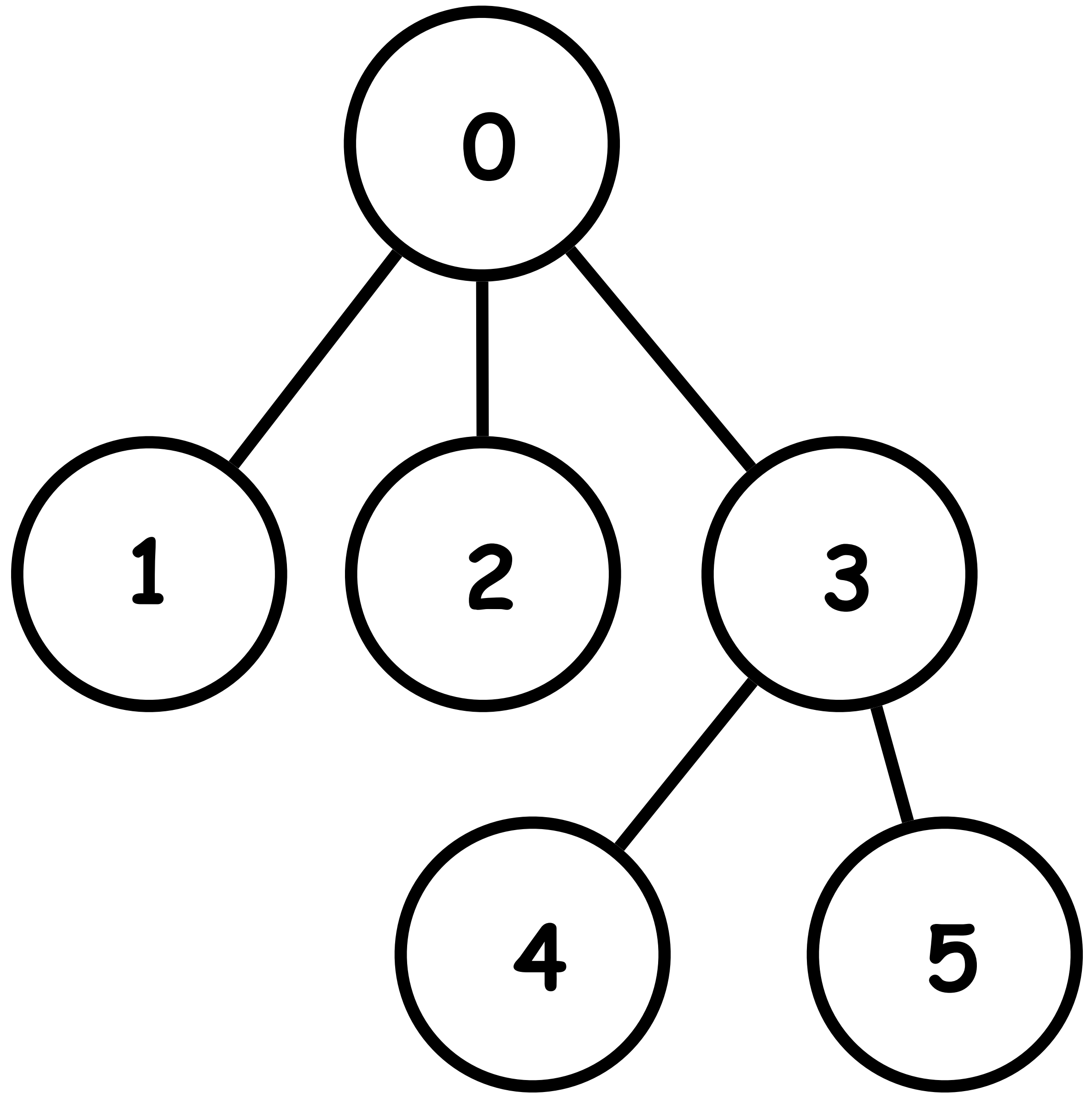

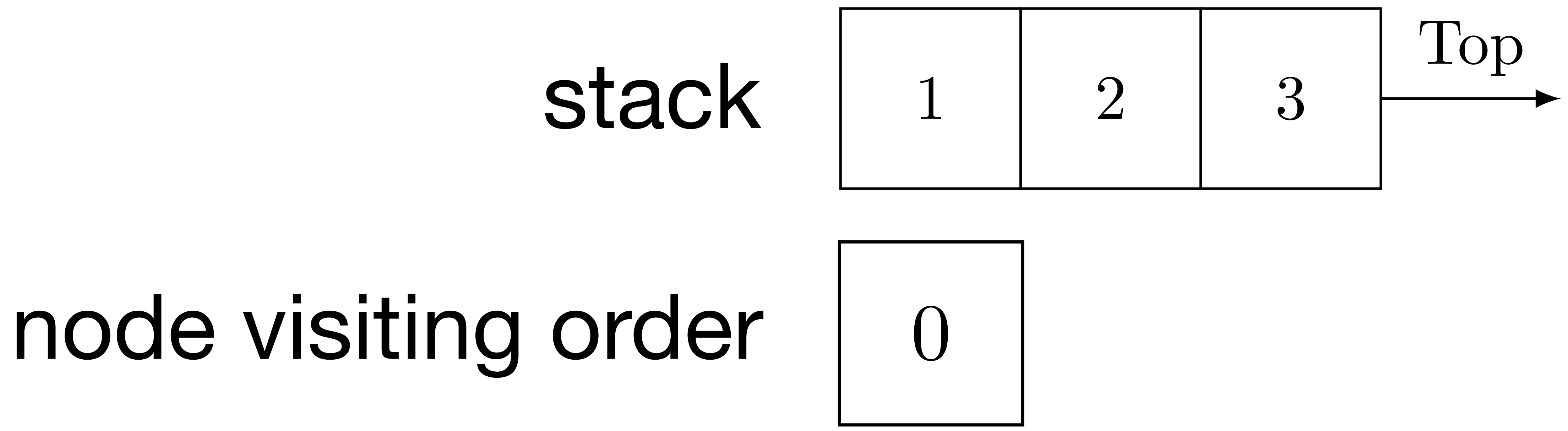

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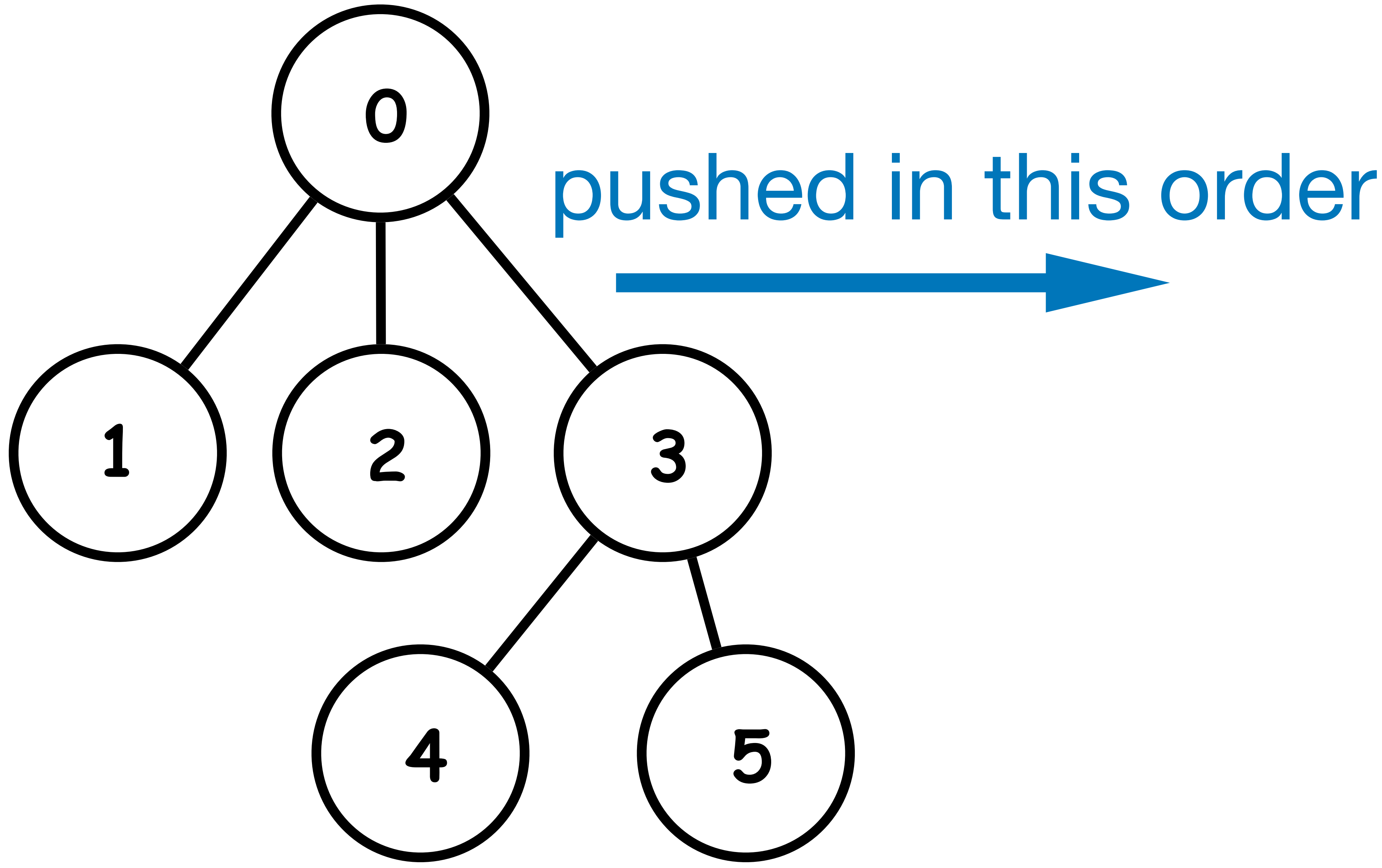

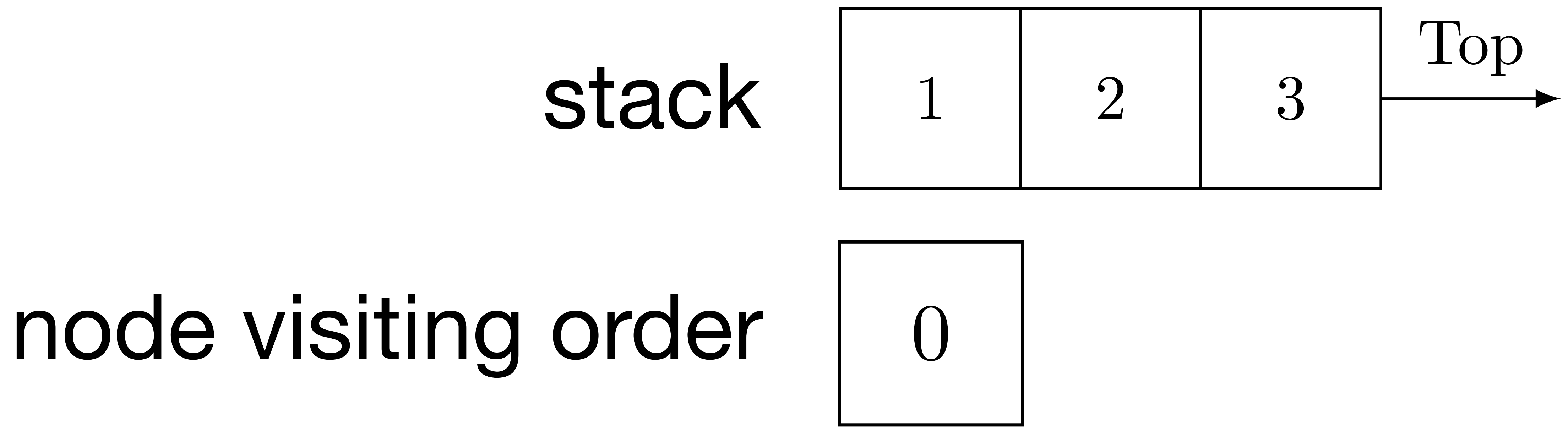

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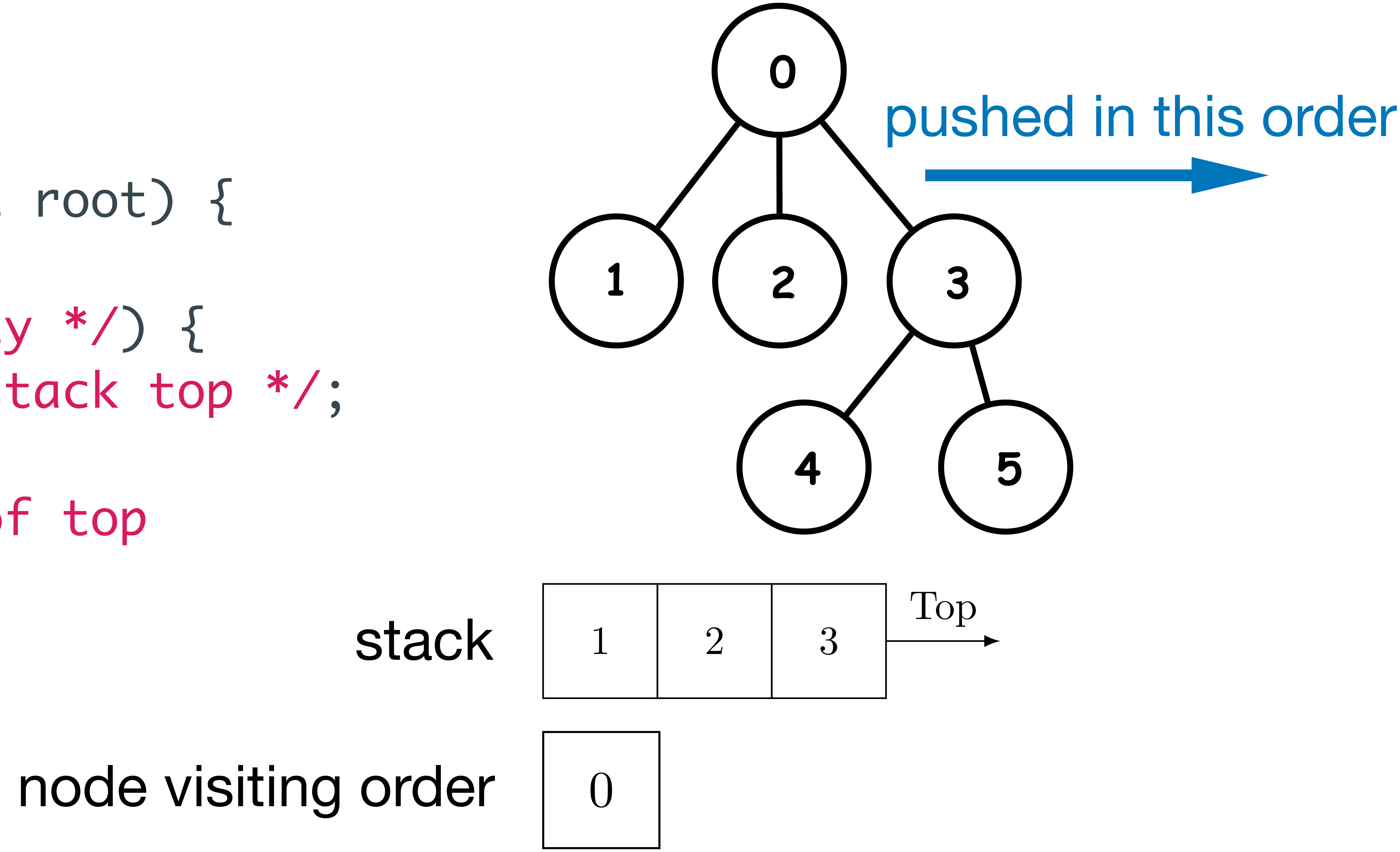

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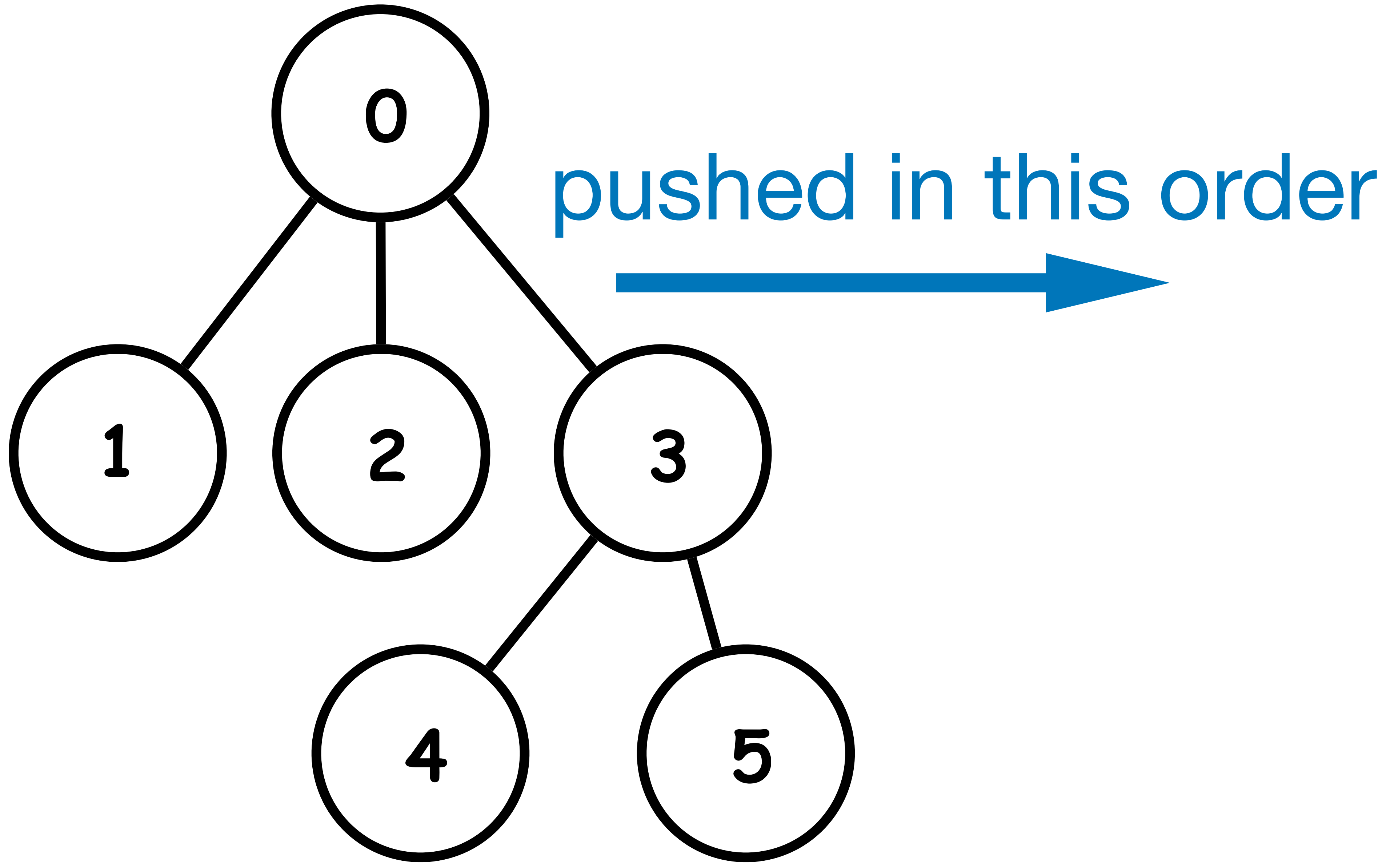

stack: 1 | 2 → Top

node visiting order: 0

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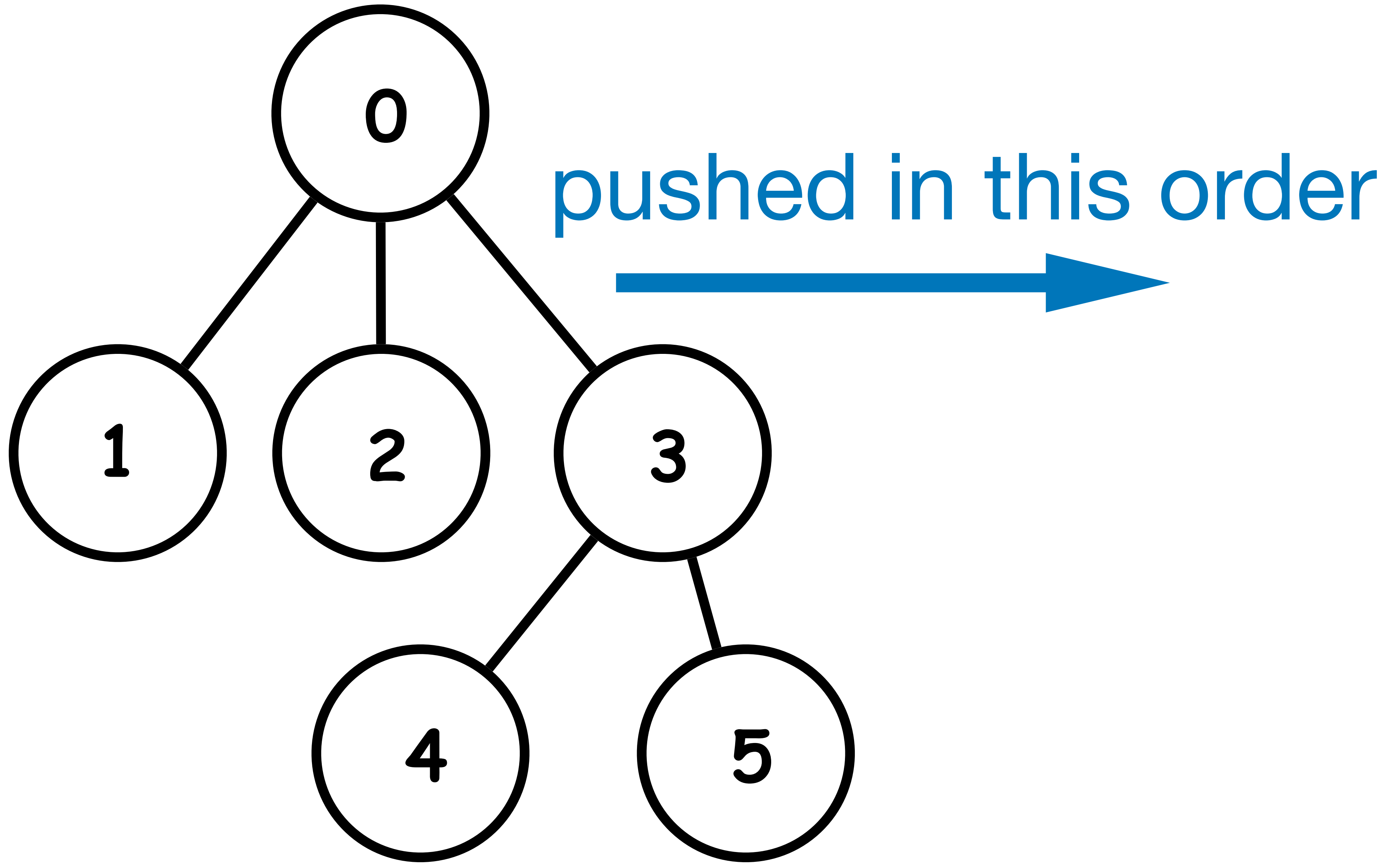

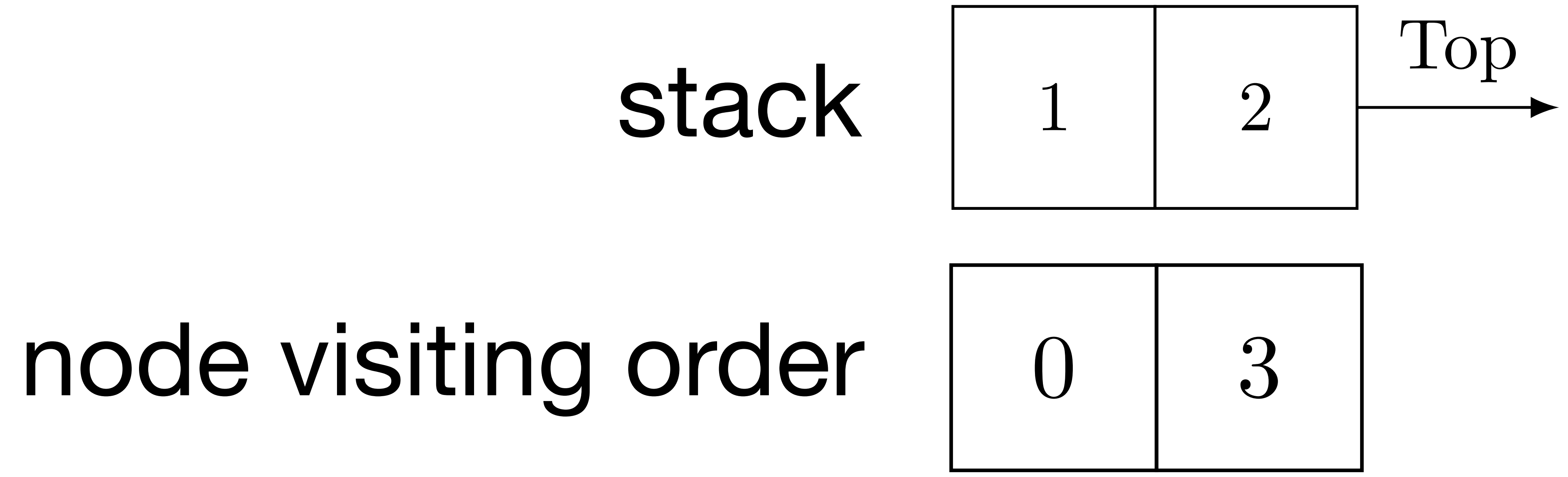

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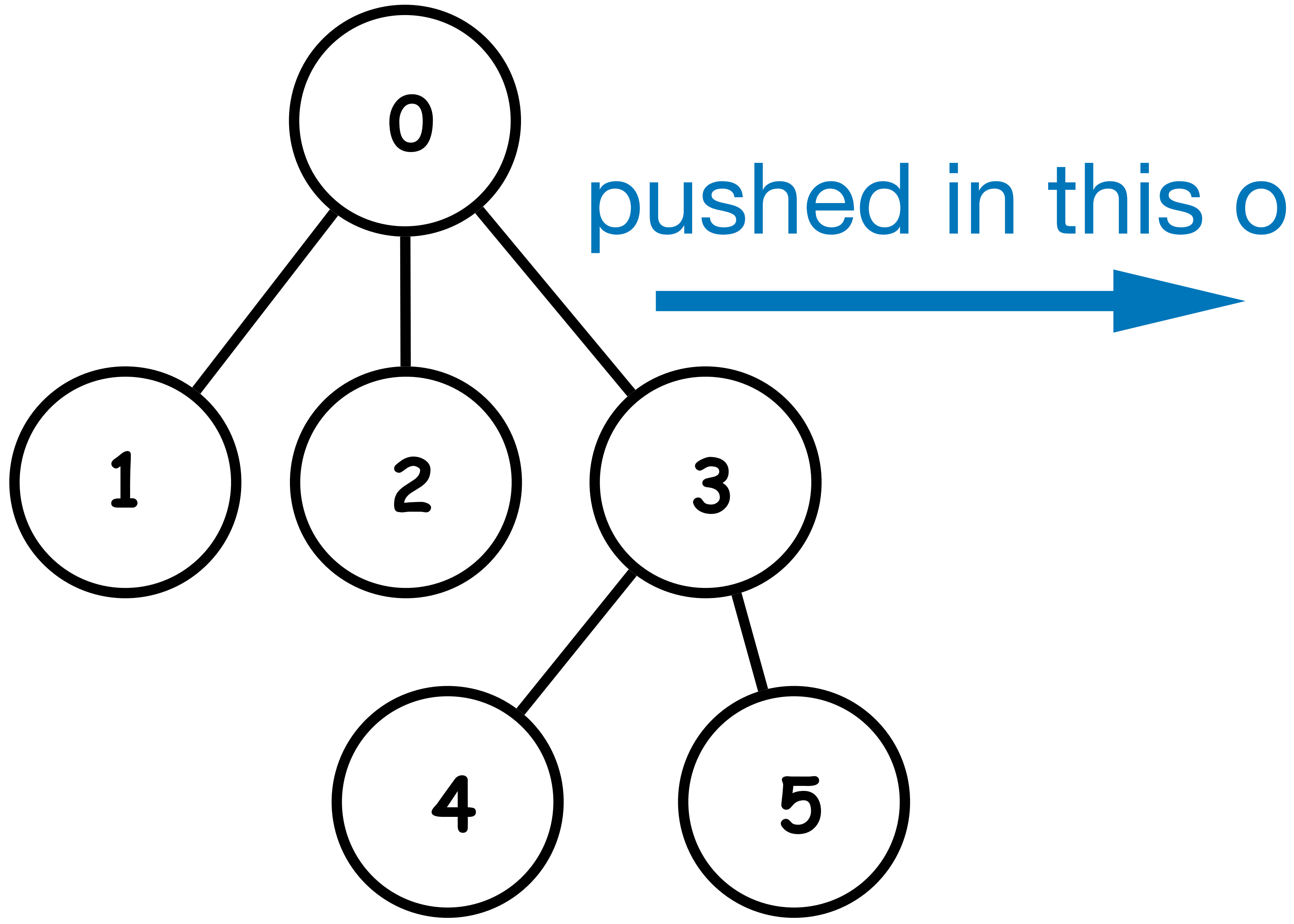

stack

1	2	4	5

Top

node visiting order

0	3

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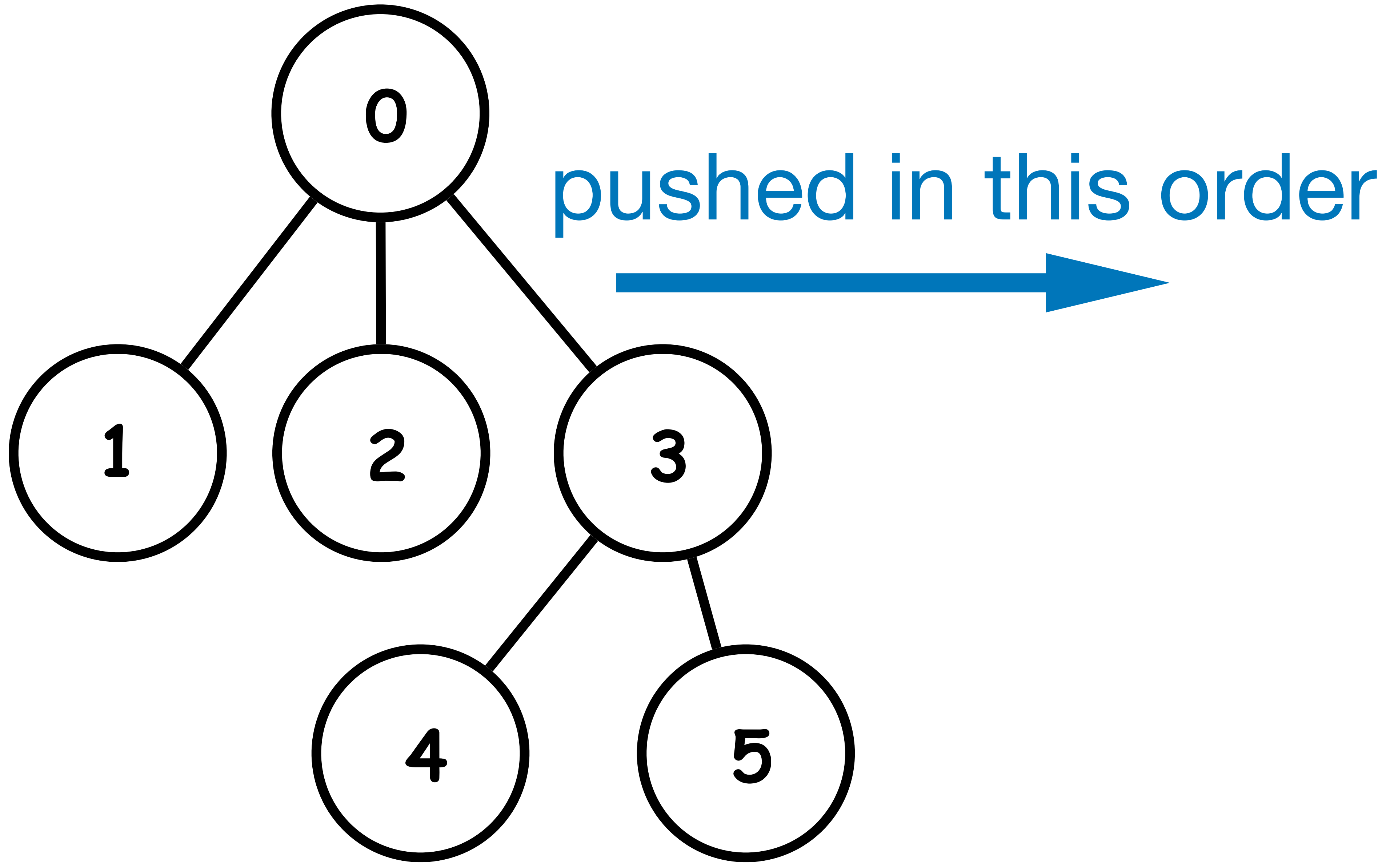

stack

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0	3	5	4	2	1

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Should be consistent & maintained in sync

Roadmap

- Challenges
- Strategies
- Case study

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Strategy to C1: Dual Views

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(defined before)

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Tree view

$$\mathrm{tree\_rep}_{\mathrm{tree}}(p, tr) \triangleq \cdots$$

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mutually derivable

switch as needed

Tree view

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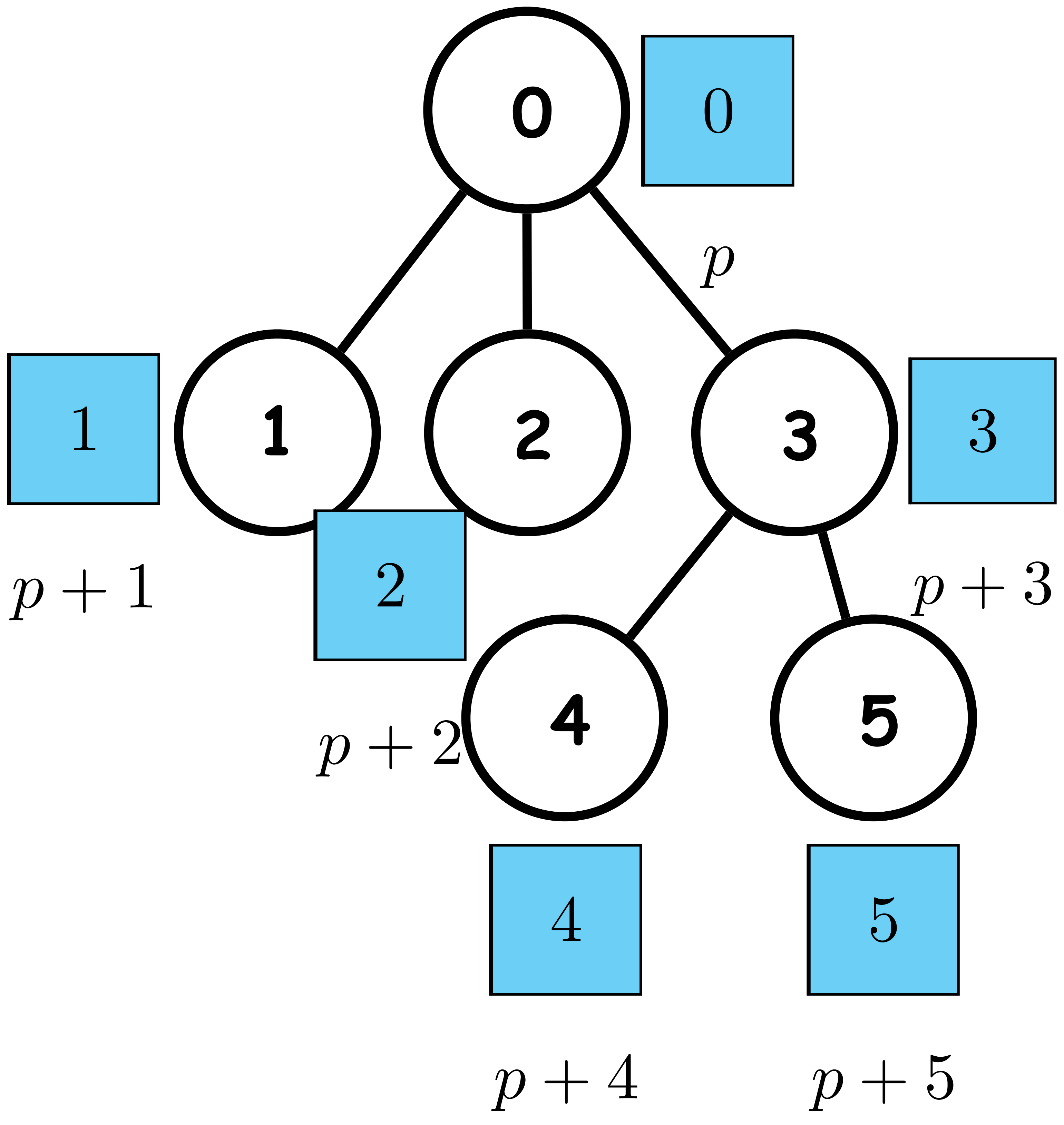

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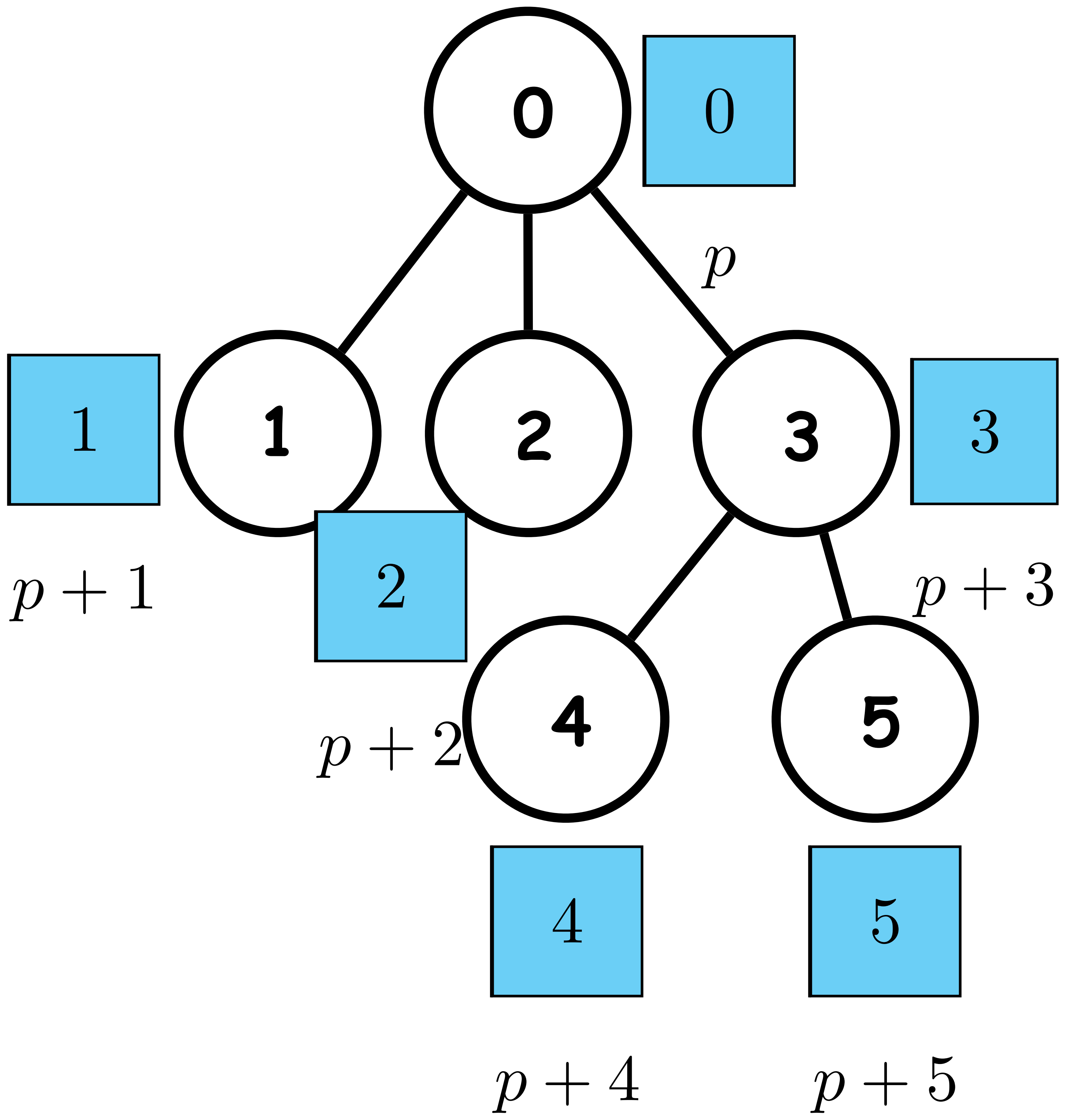
$$\text{tree\_rep}_{\text{tree}}(p, tr) = \cdots * \\ p + 3 \mapsto \{\texttt{parent} = 0, \texttt{right\_sibling} = -1, \\ \texttt{first\_child} = 4\} * \cdots$$



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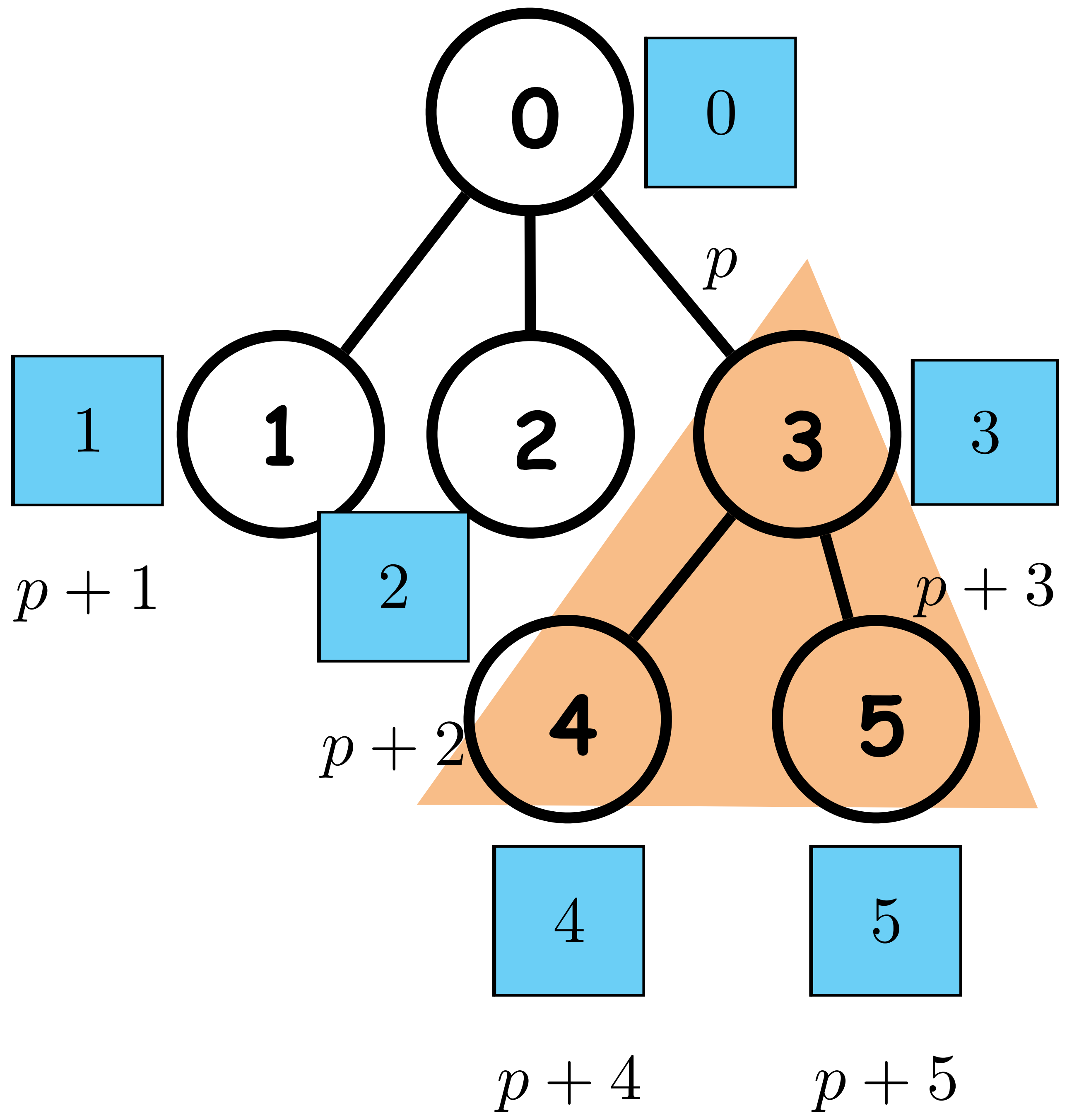
- E.g., "focusing" on a subtree



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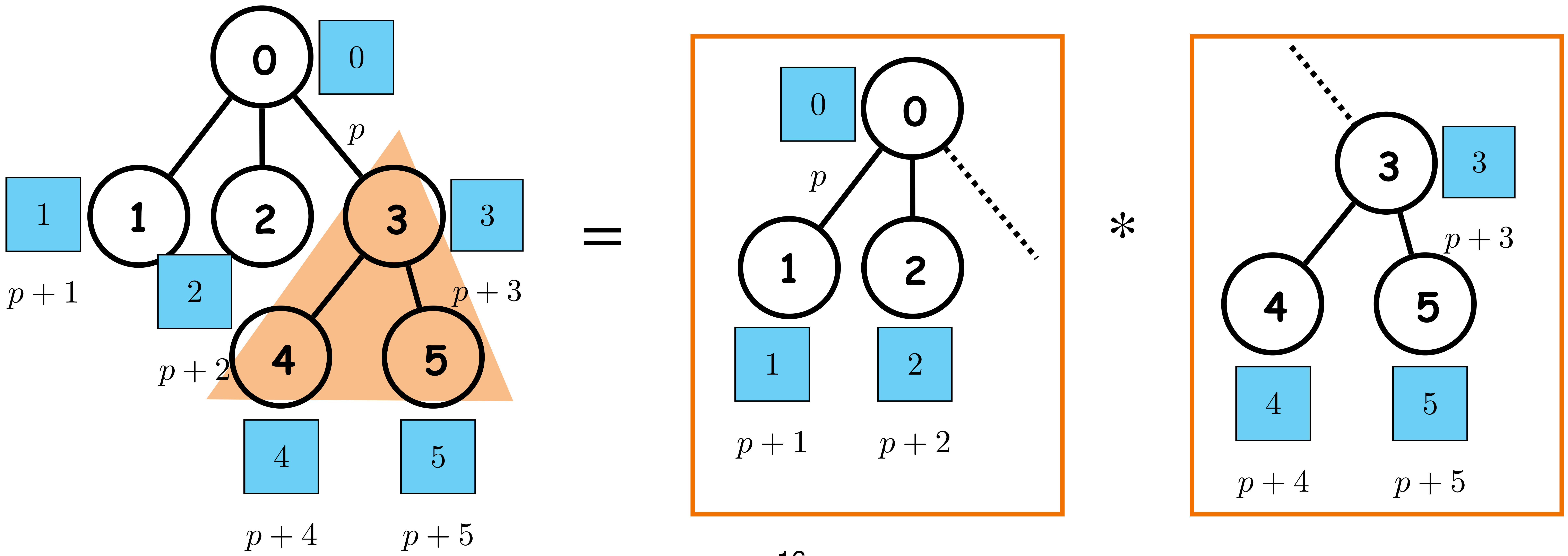

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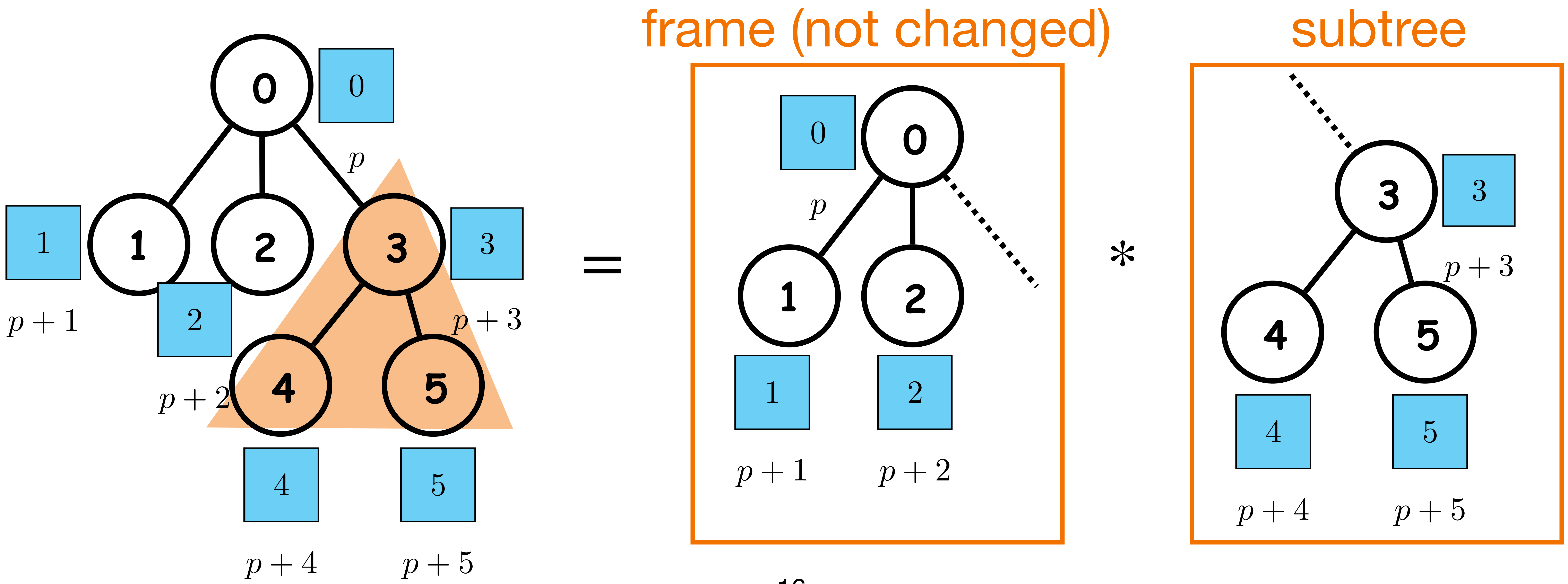

Mutual Derivability of Dual Views

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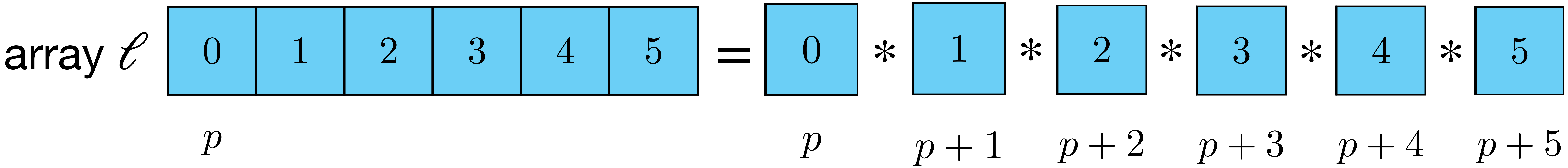

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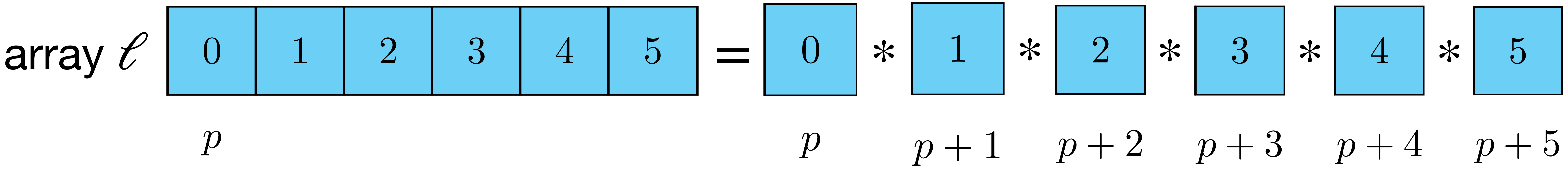

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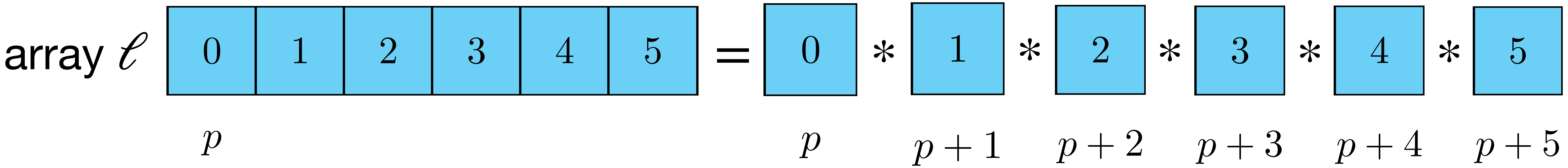

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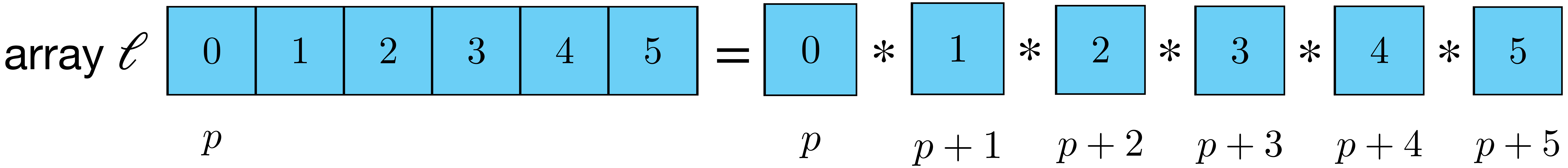

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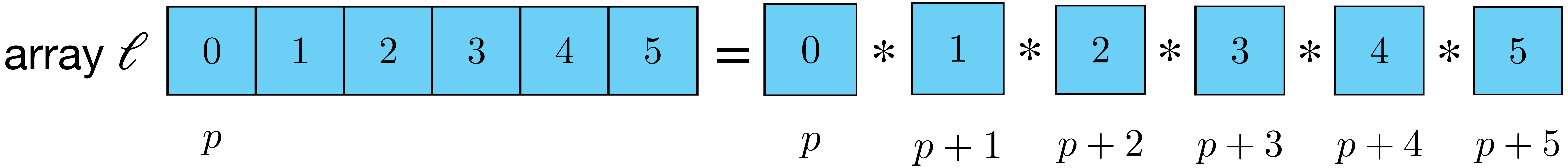

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mutually derivable!

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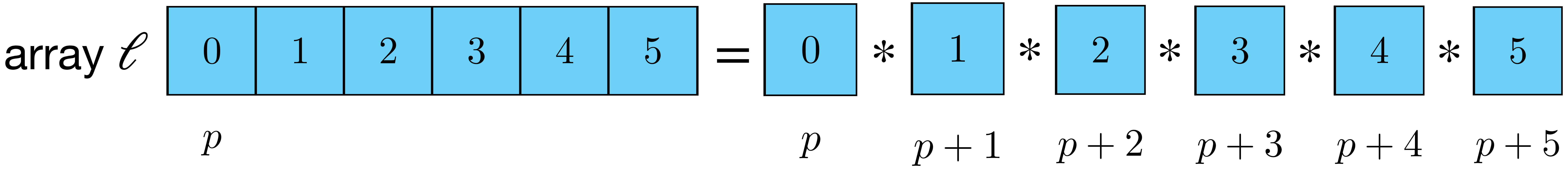

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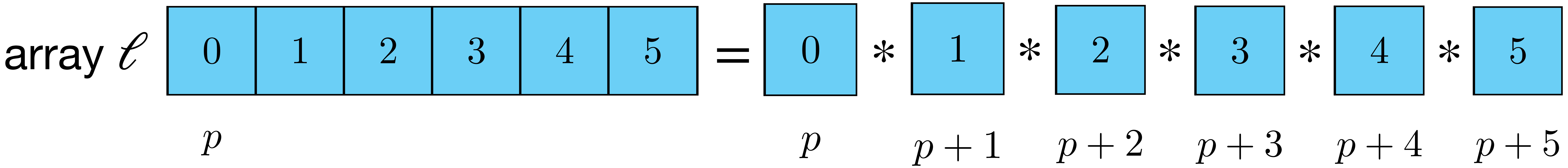

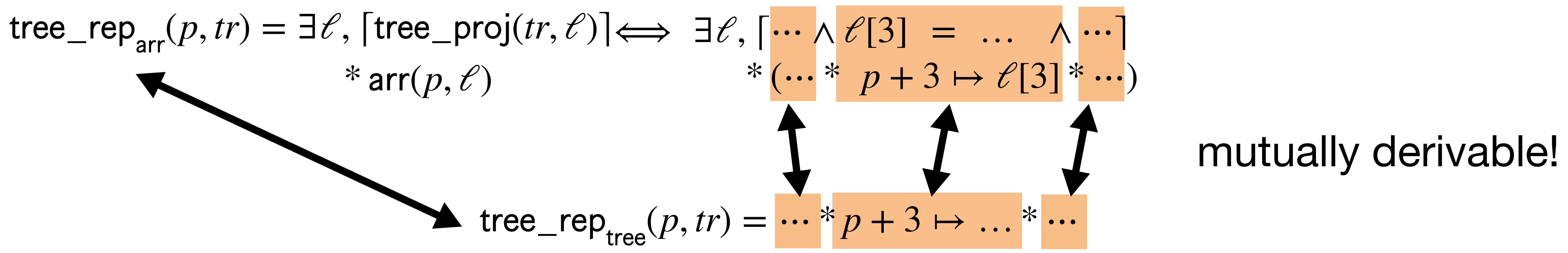

Dual Views in Action

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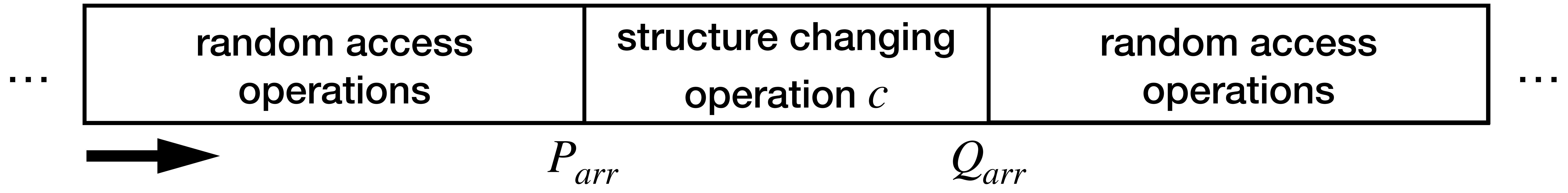

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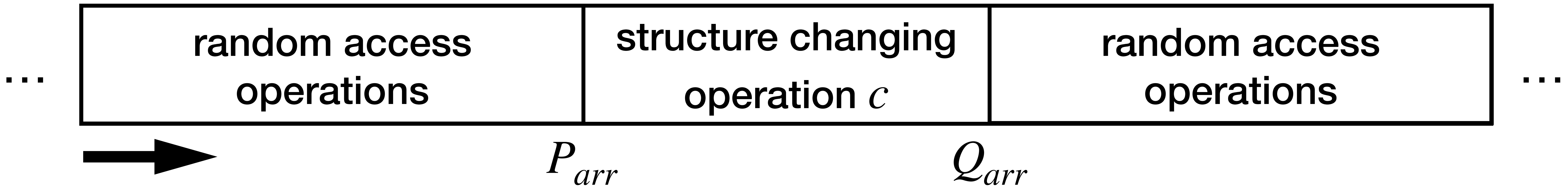

$$\frac{P_{arr} \vdash P_{tree} \qquad \{P_{tree}\}\ c\ \{Q_{tree}\} \qquad Q_{tree} \vdash Q_{arr}}{\{P_{arr}\}\ c\ \{Q_{arr}\}}$$

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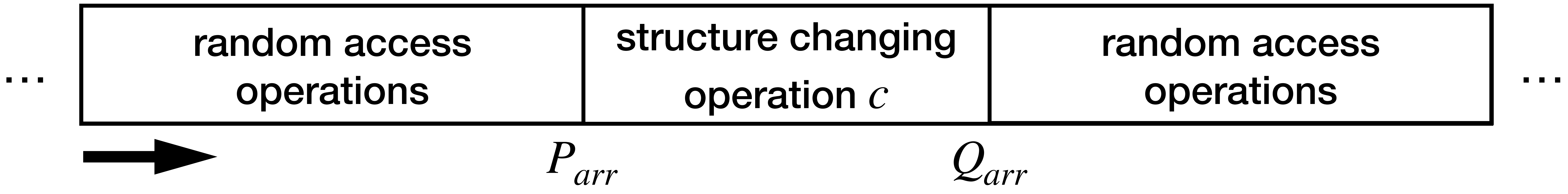

tree_rep$_{\text{arr}}(p, tr) \vdash$ tree_rep$_{\text{tree}}(p, tr)$

tree_rep$_{\text{tree}}(p, tr') \vdash$ tree_rep$_{\text{arr}}(p, tr')$

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Dual Views in Action

… | random access operations | structure changing operation c | random access operations | …

P_{arr} $\quad$ Q_{arr}

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    - The visited part: expressed as the right half of tree splitting

Vertical Split

Reminder: children are pushed from left to right

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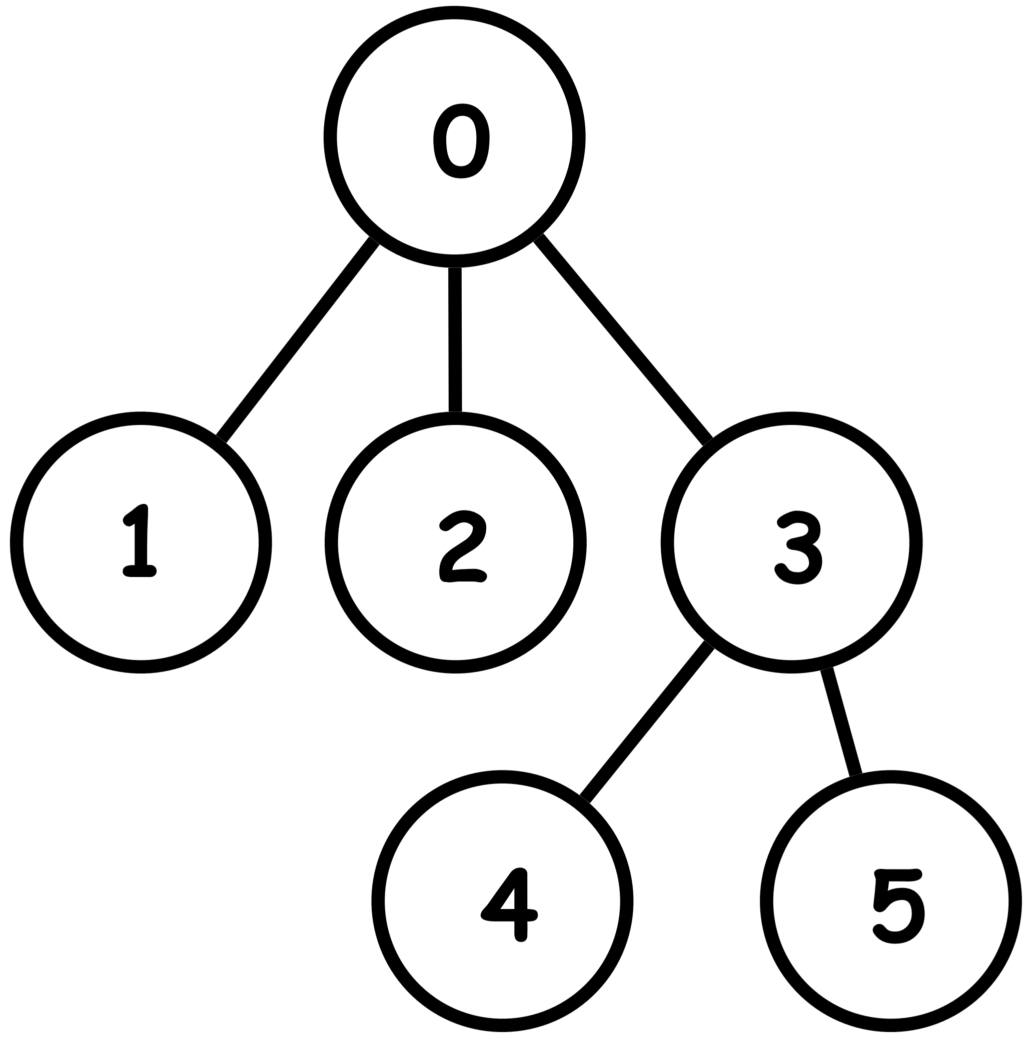

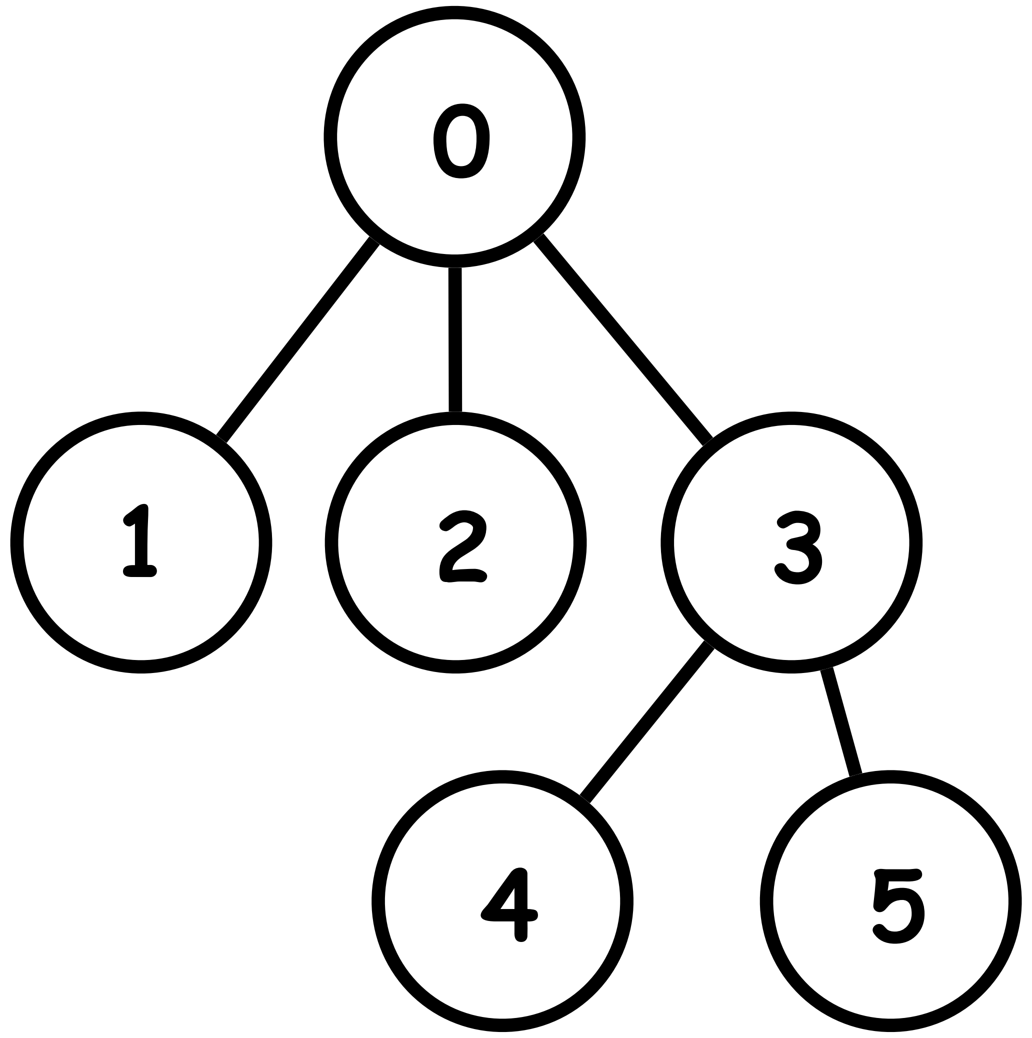

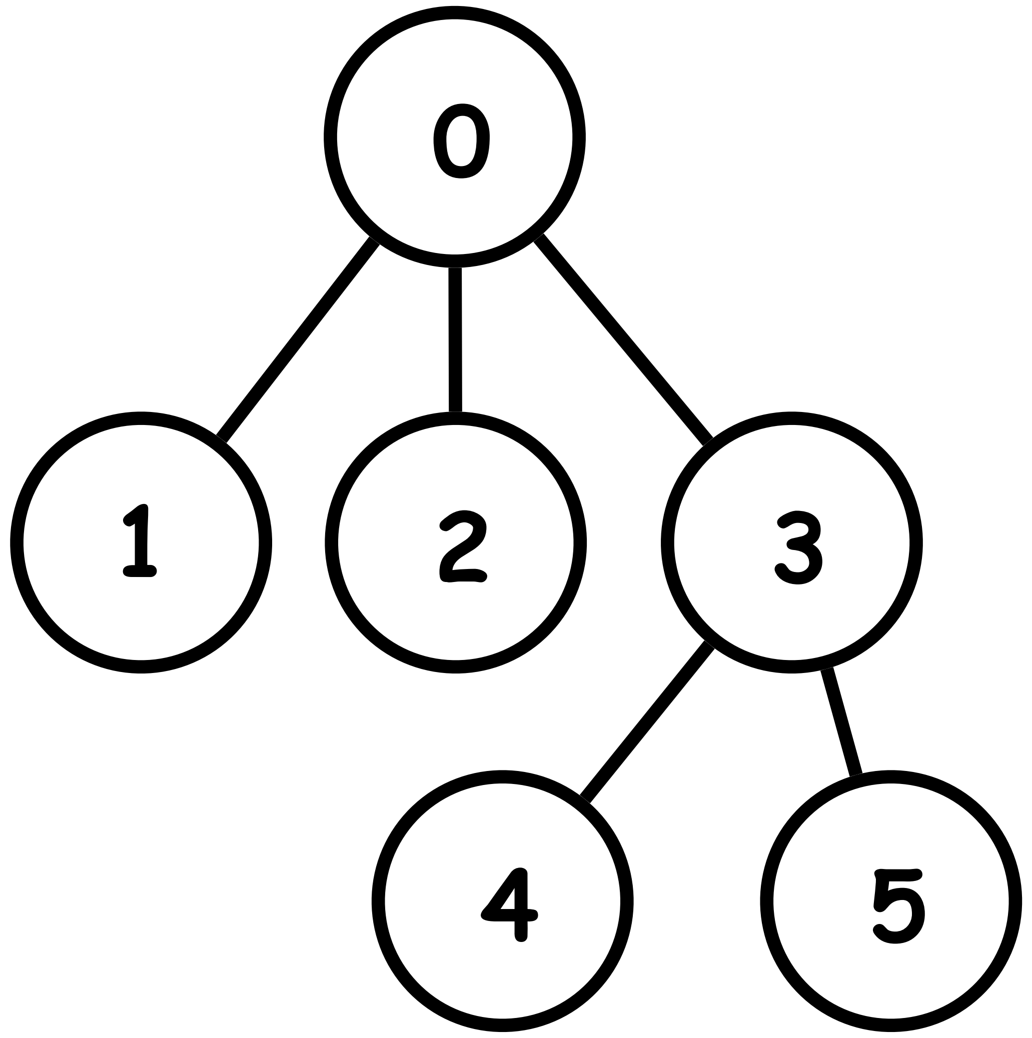

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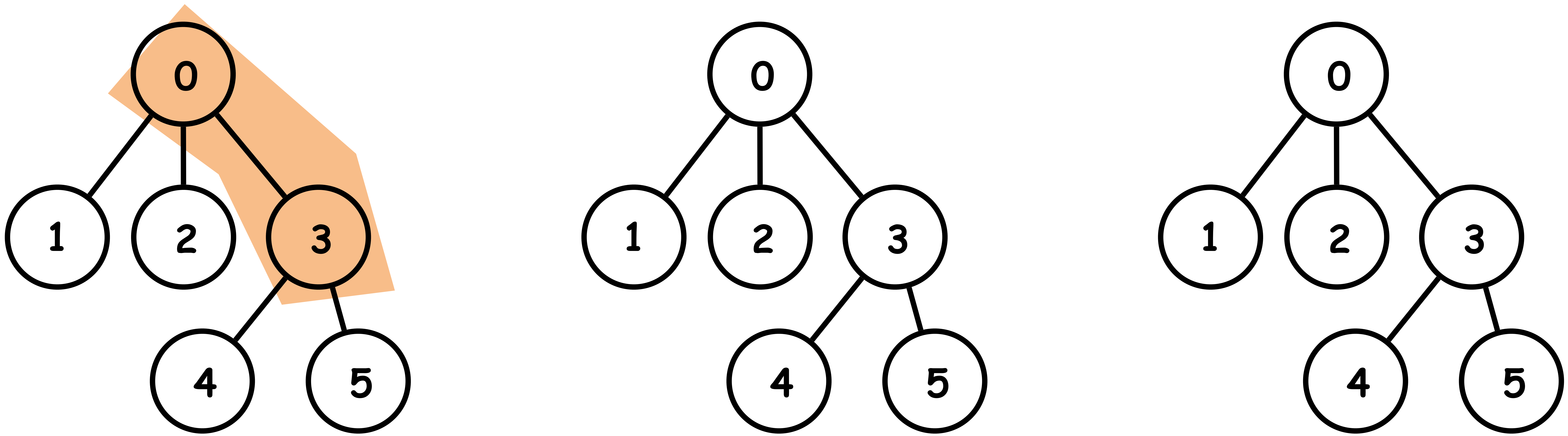

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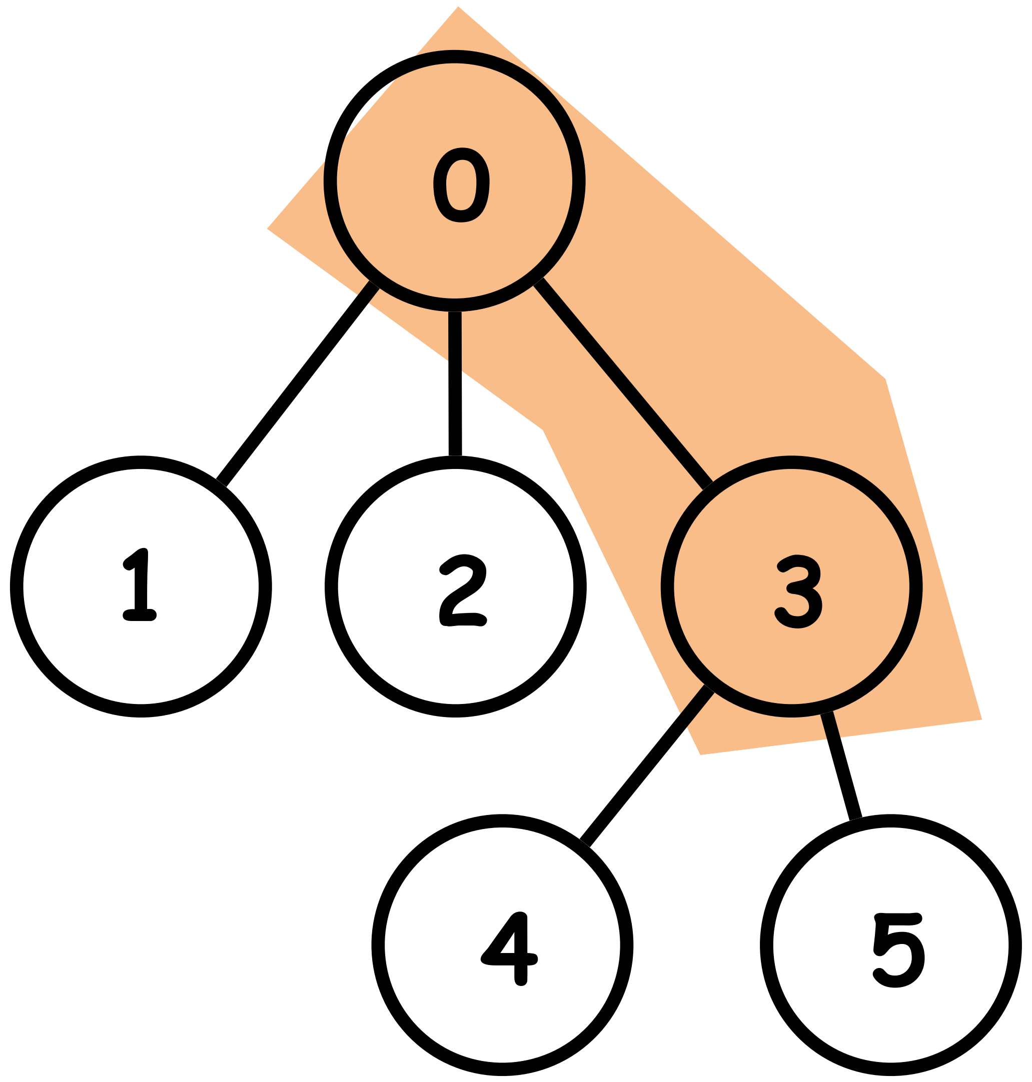

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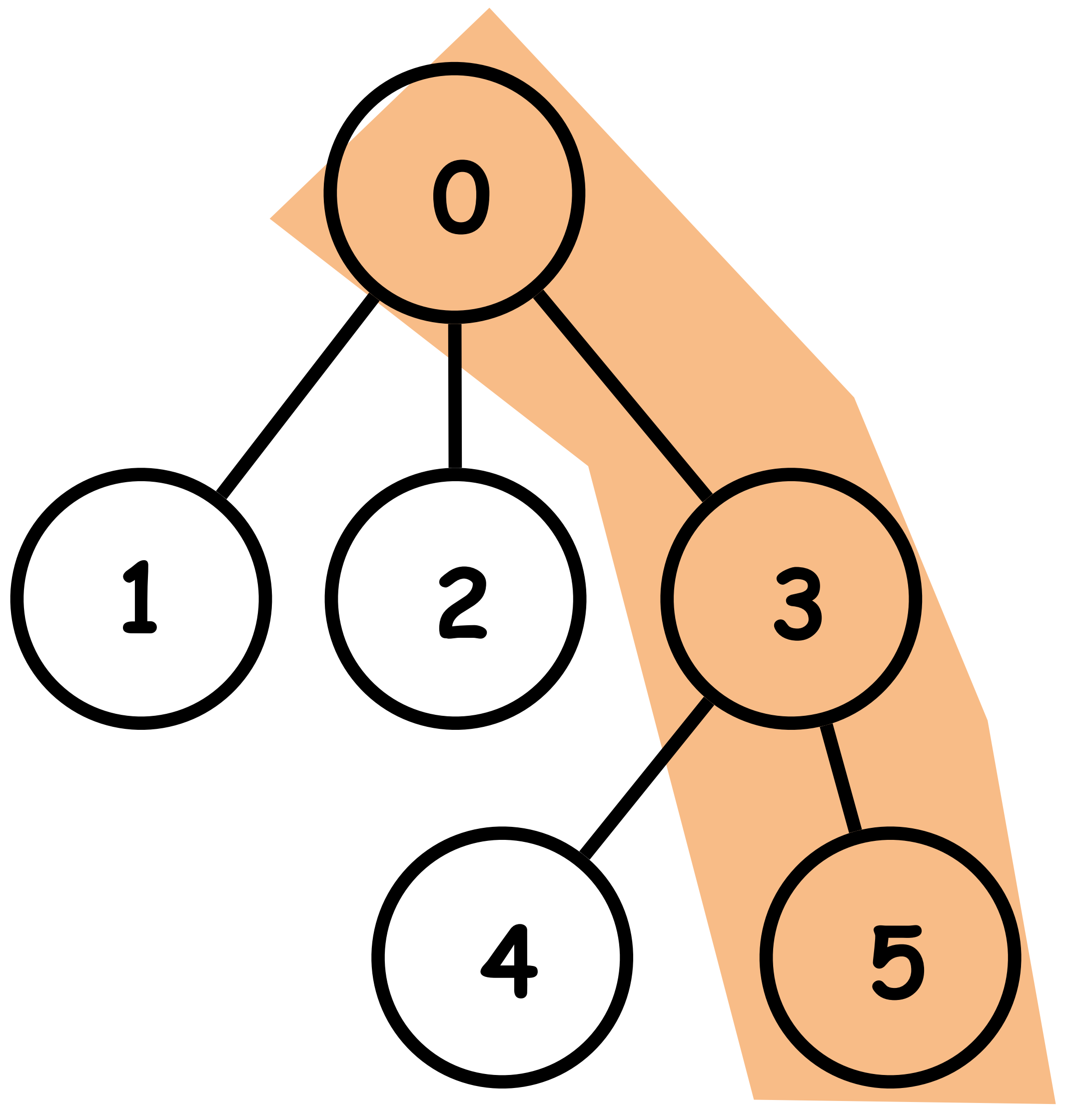
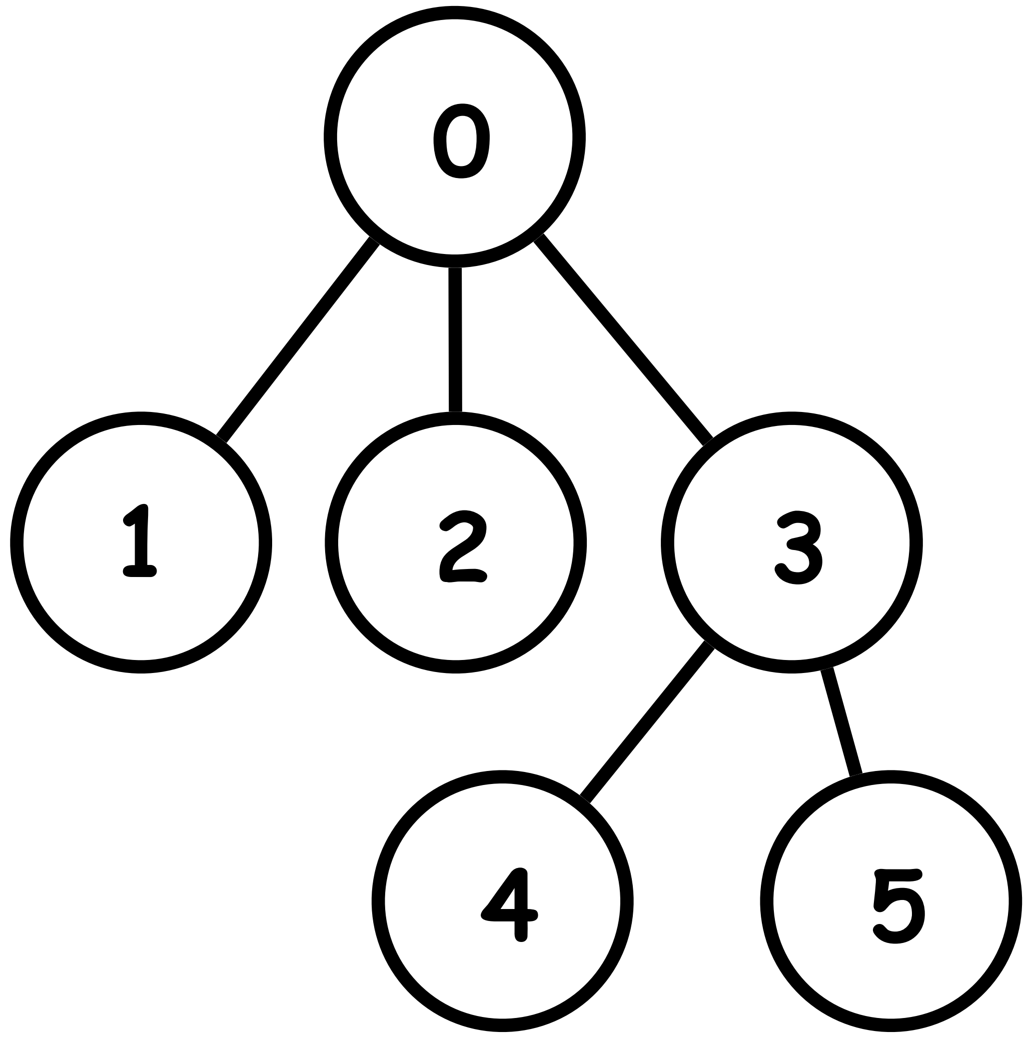


split by node 5



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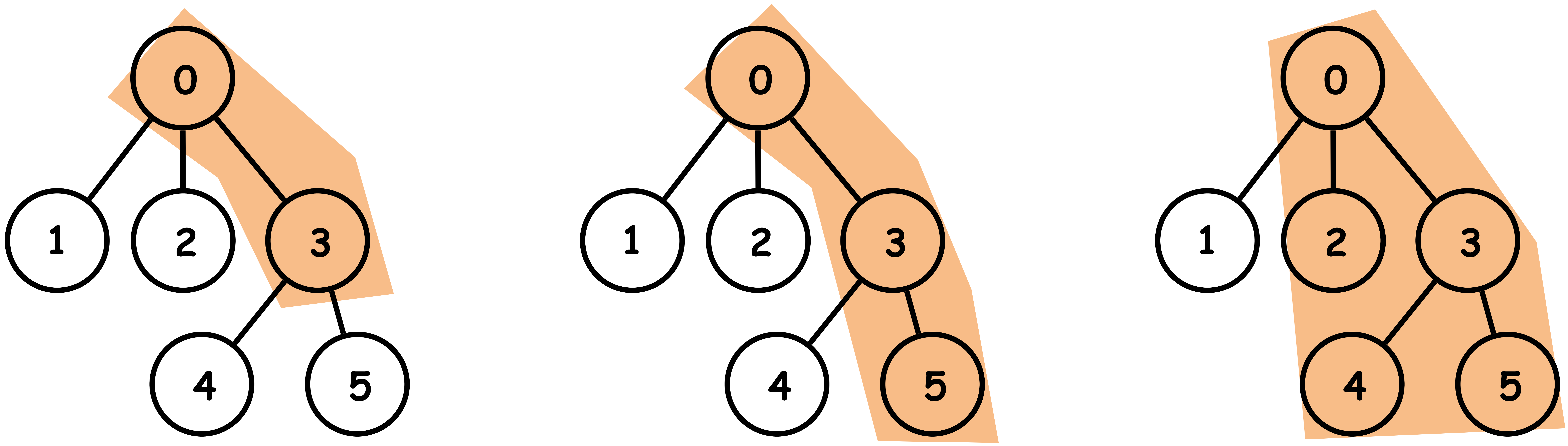

split by node 3 split by node 5 split by node 2

Vertical Split

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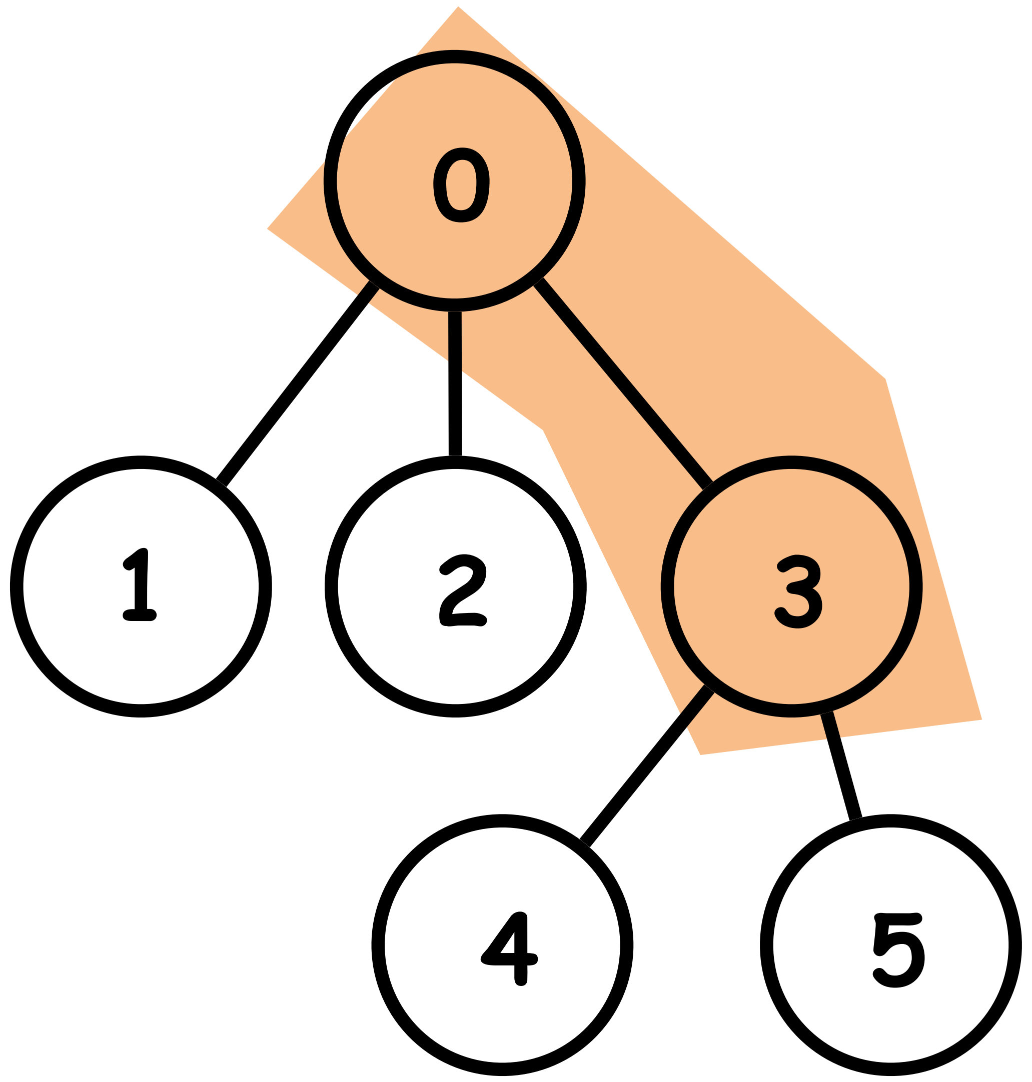
- Split the tree vertically along the path from the root to a node
  - The right half: including that node, its ancestors and the subtrees on their right



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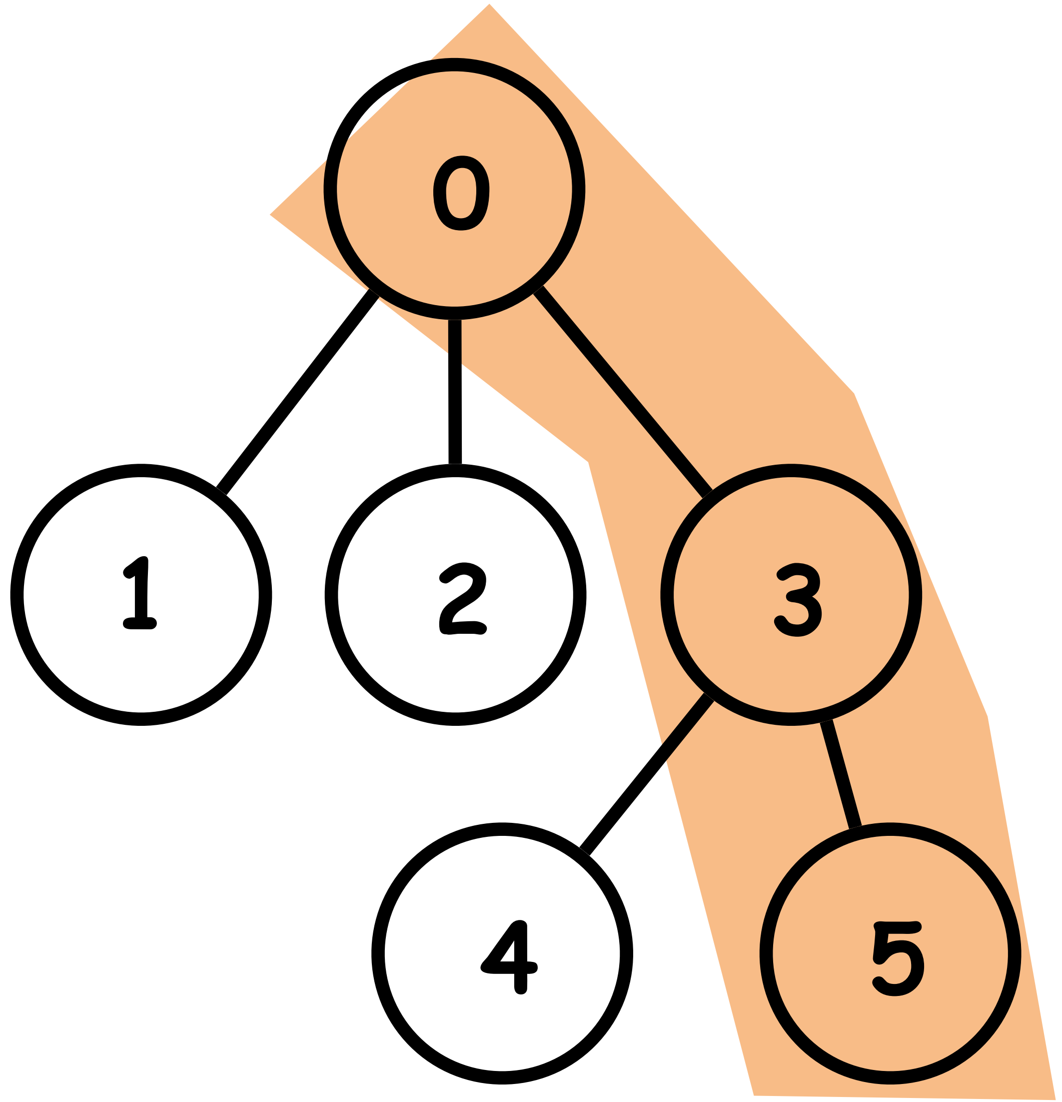

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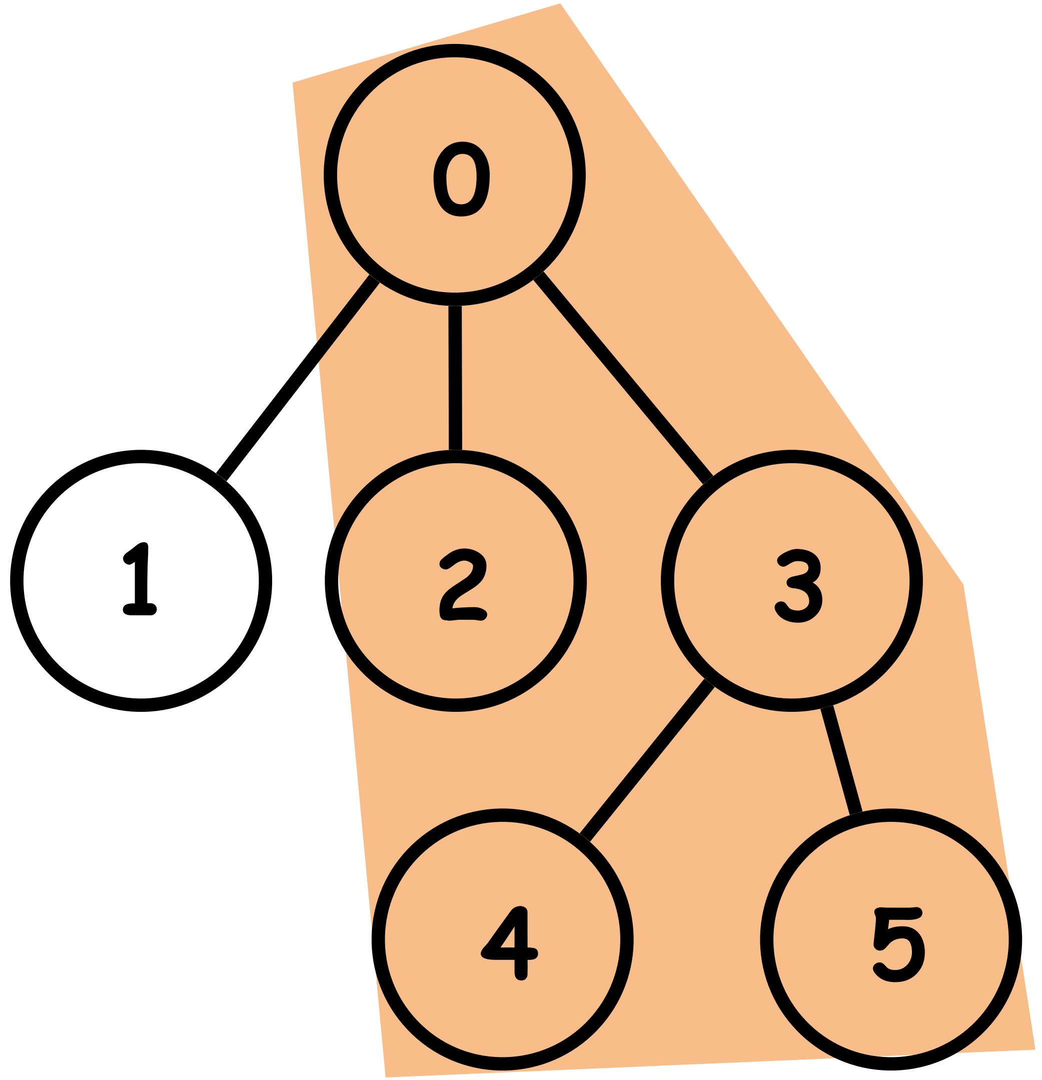

split by node 2

Vertical Split and Visited Part

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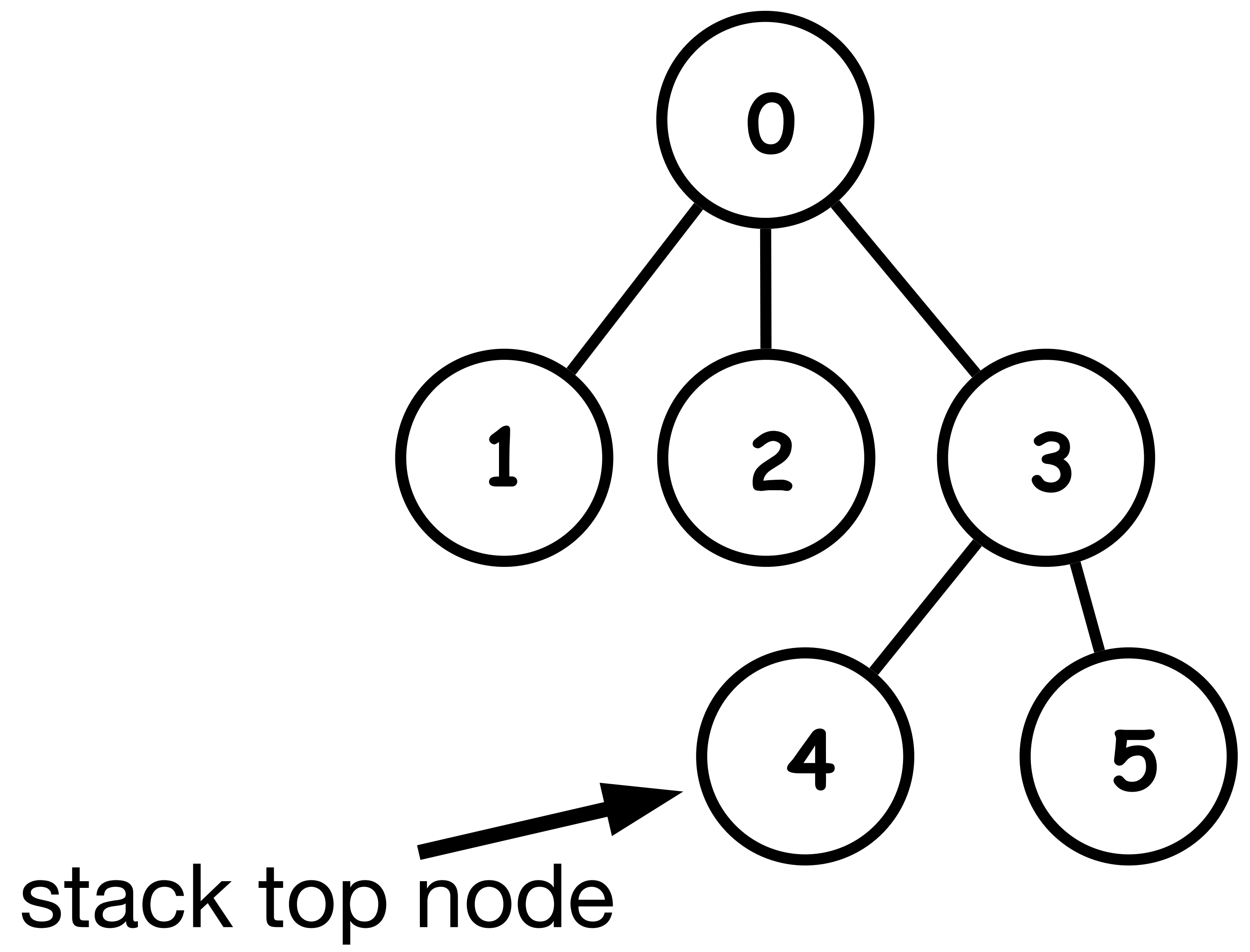

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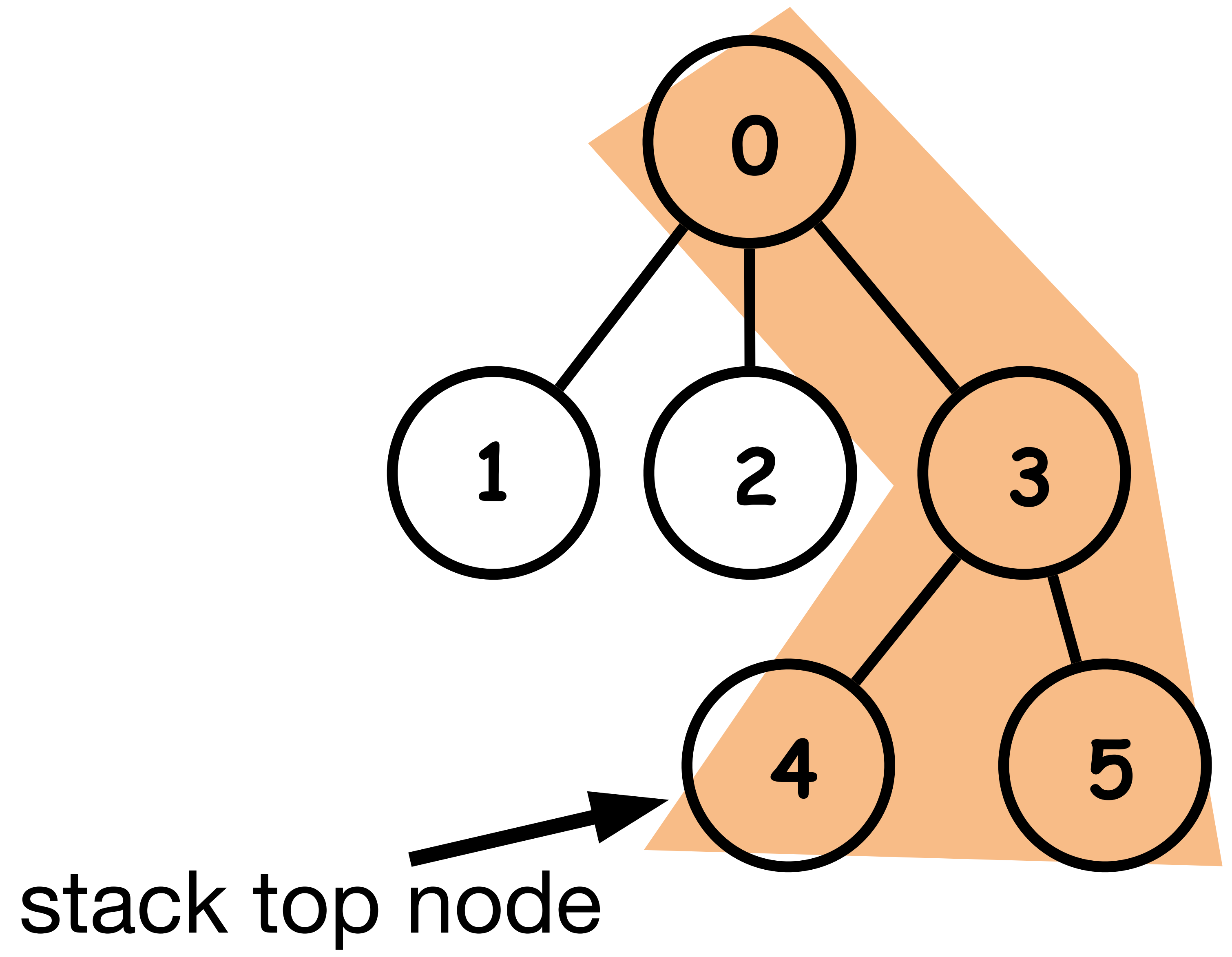

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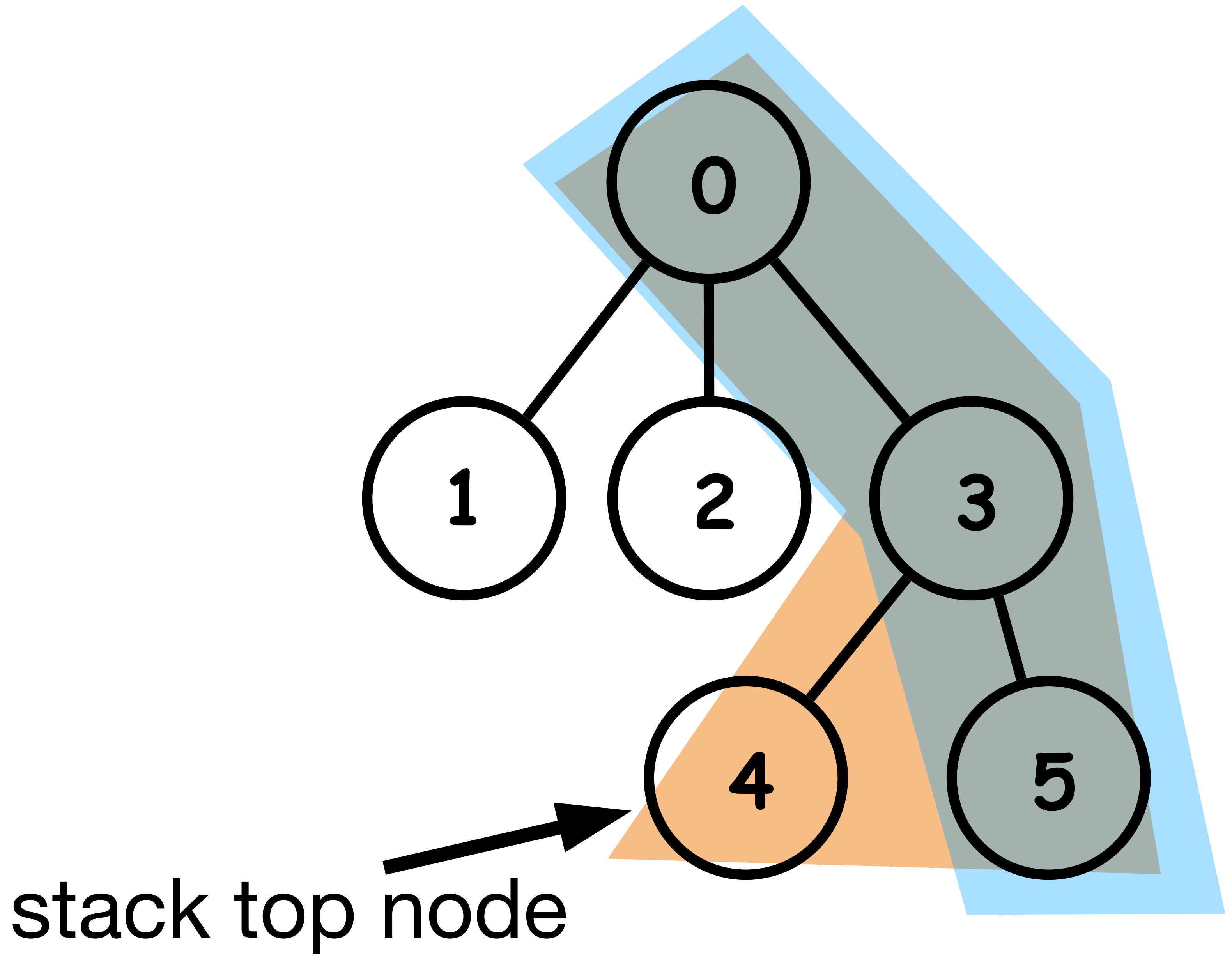

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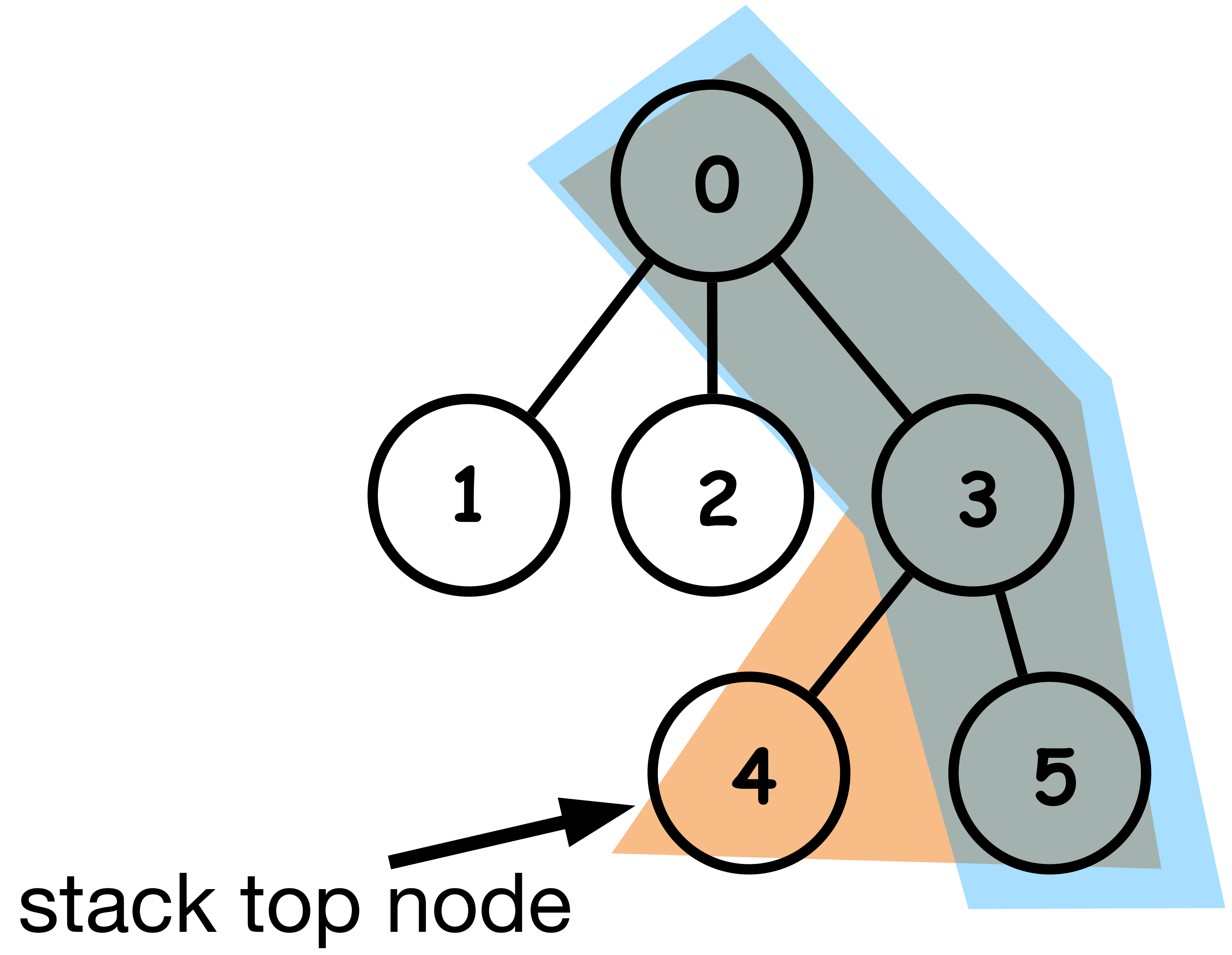

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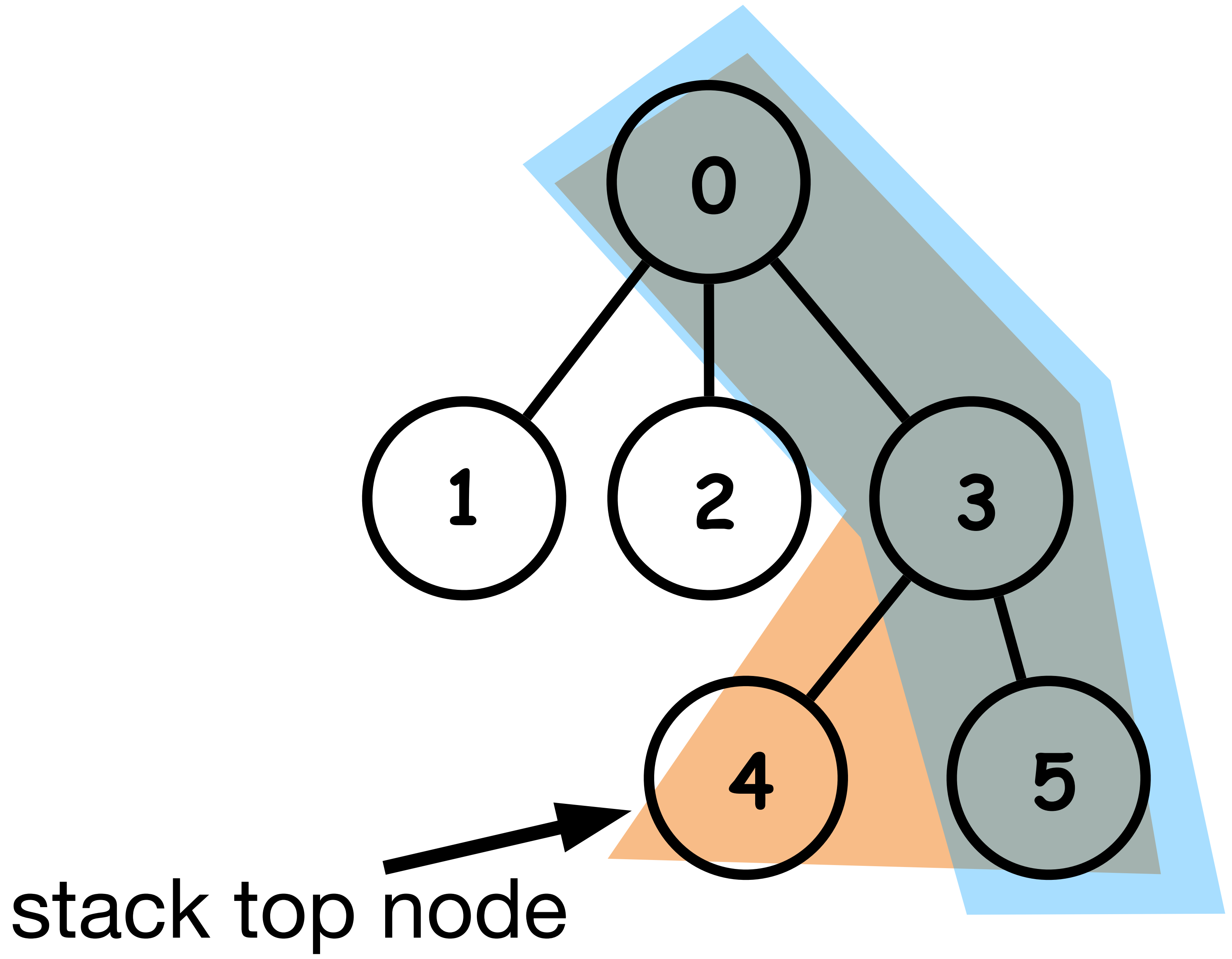

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Check our paper for exact definition!

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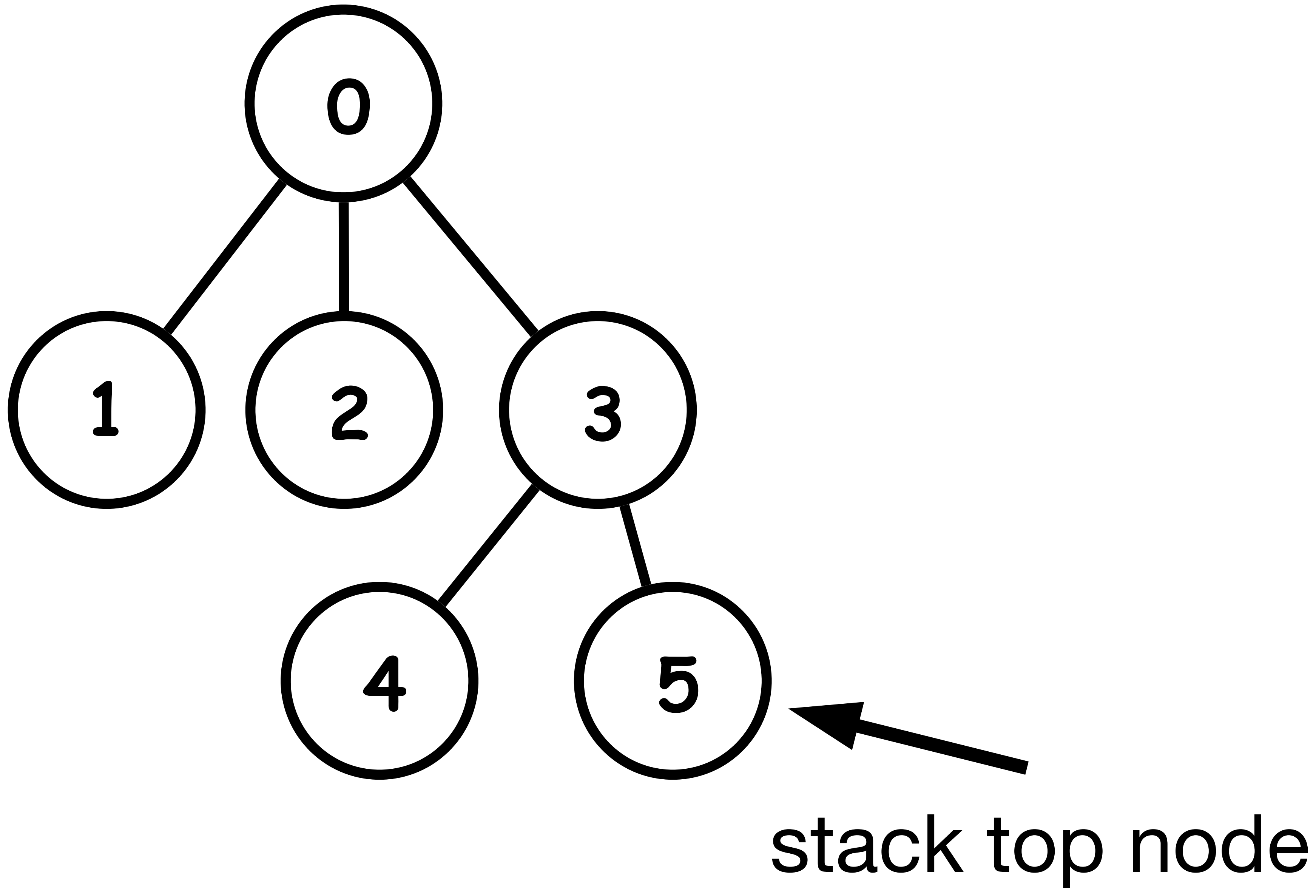

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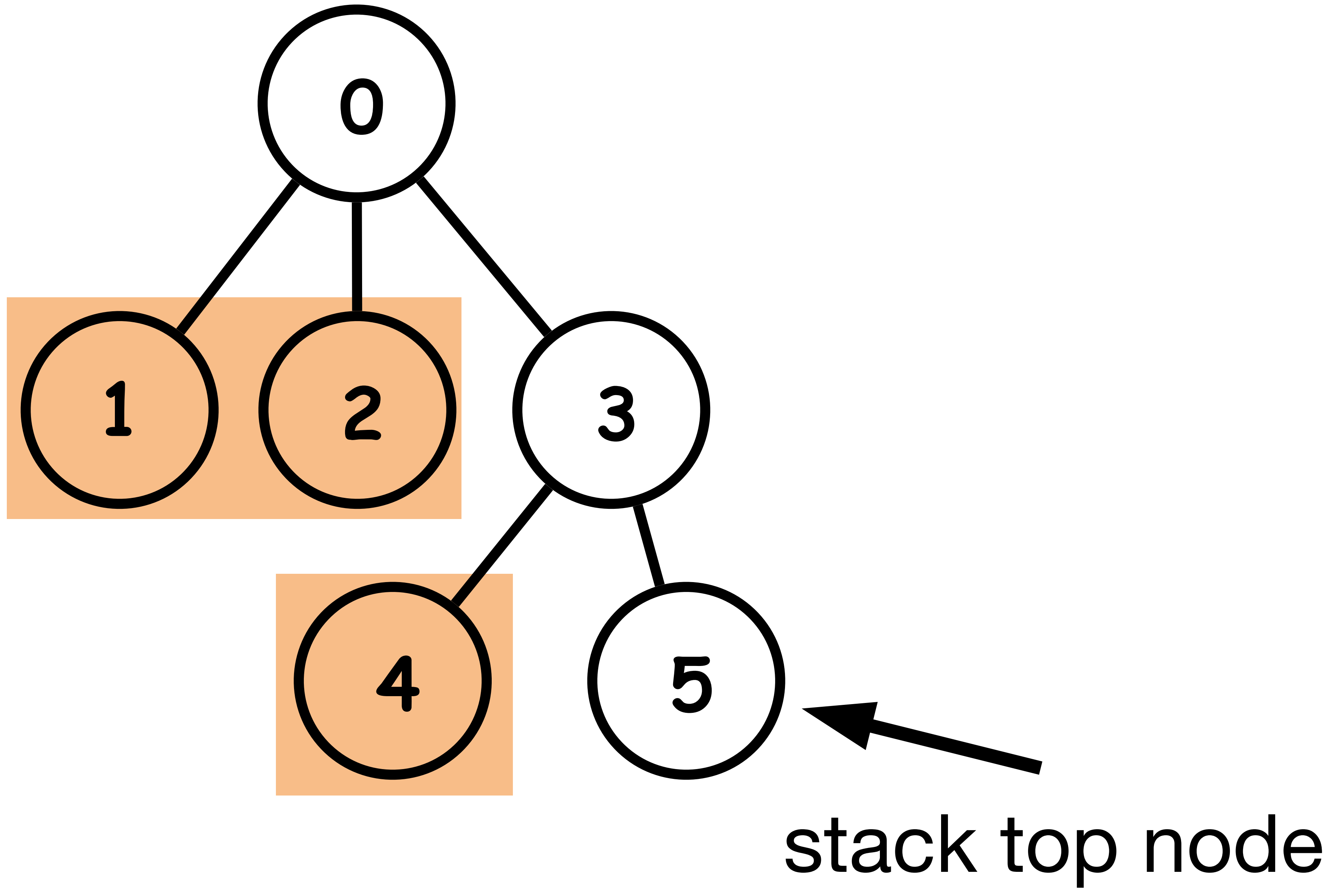

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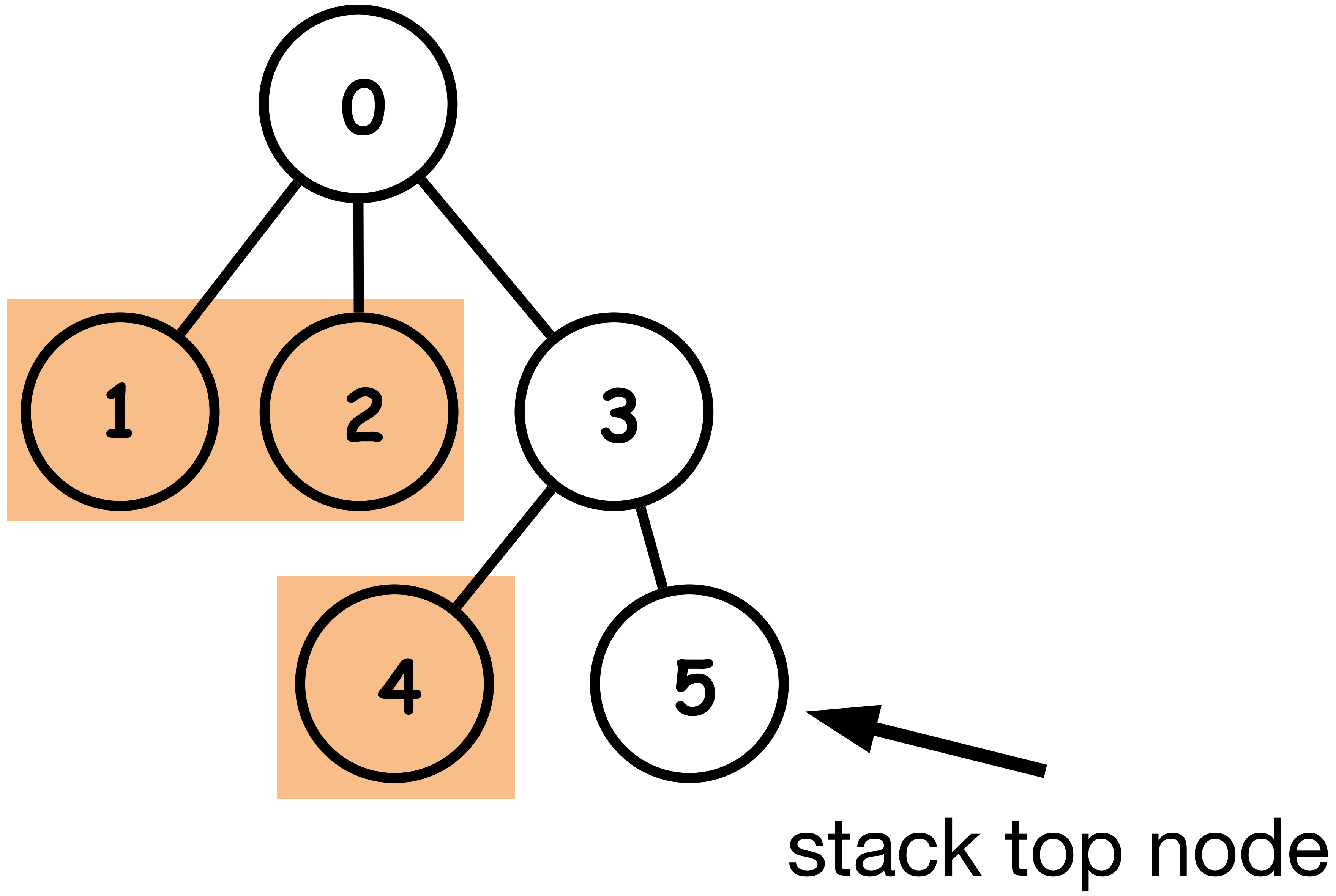

- The stack before an iteration = nodes on the left of the ancestors of stack top
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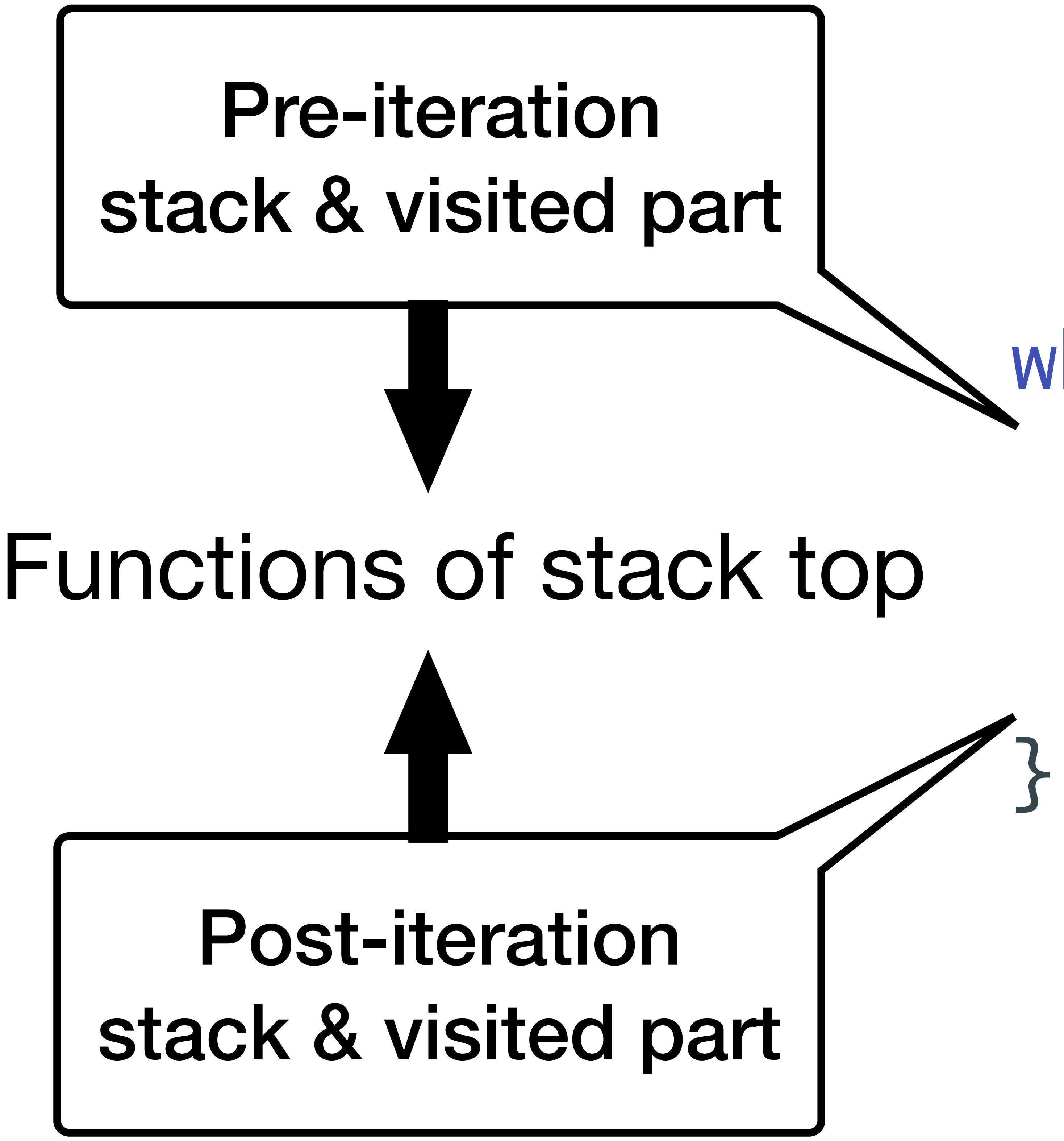
Loop Invariant of Non-Recursive Traversal

- Sufficient to define by keeping track of the stack top node

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Roadmap

- Challenges
- Strategies
- Case study

(published in ASPLOS 2022)

A Tree Clock Data Structure for Causal Orderings in Concurrent Executions

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ABSTRACT

Dynamic techniques are a scalable and effective way to analyze concurrent programs. Instead of analyzing all behaviors of a program, these techniques detect errors by focusing on a single program execution. Often a crucial step in these techniques is to define a causal ordering between events in the execution, which is then computed using *vector clocks*, a simple data structure that stores logical times of threads. The two basic operations of vector clocks, namely join and copy, require $\Theta(k)$ time, where k is the number of threads. Thus they are a computational bottleneck when k is large.

In this work, we introduce *tree clocks*, a new data structure that re-

KEYWORDS

concurrency, happens-before, vector clocks, dynamic analyses

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Tree Clock

- Implementing logical clocks using generic trees

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$$(t_1 : 16,\ t_2 : 20,\ t_3 : 17,$$
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vector clock

Tree Clock

- Implementing <u>logical clocks</u> using generic trees

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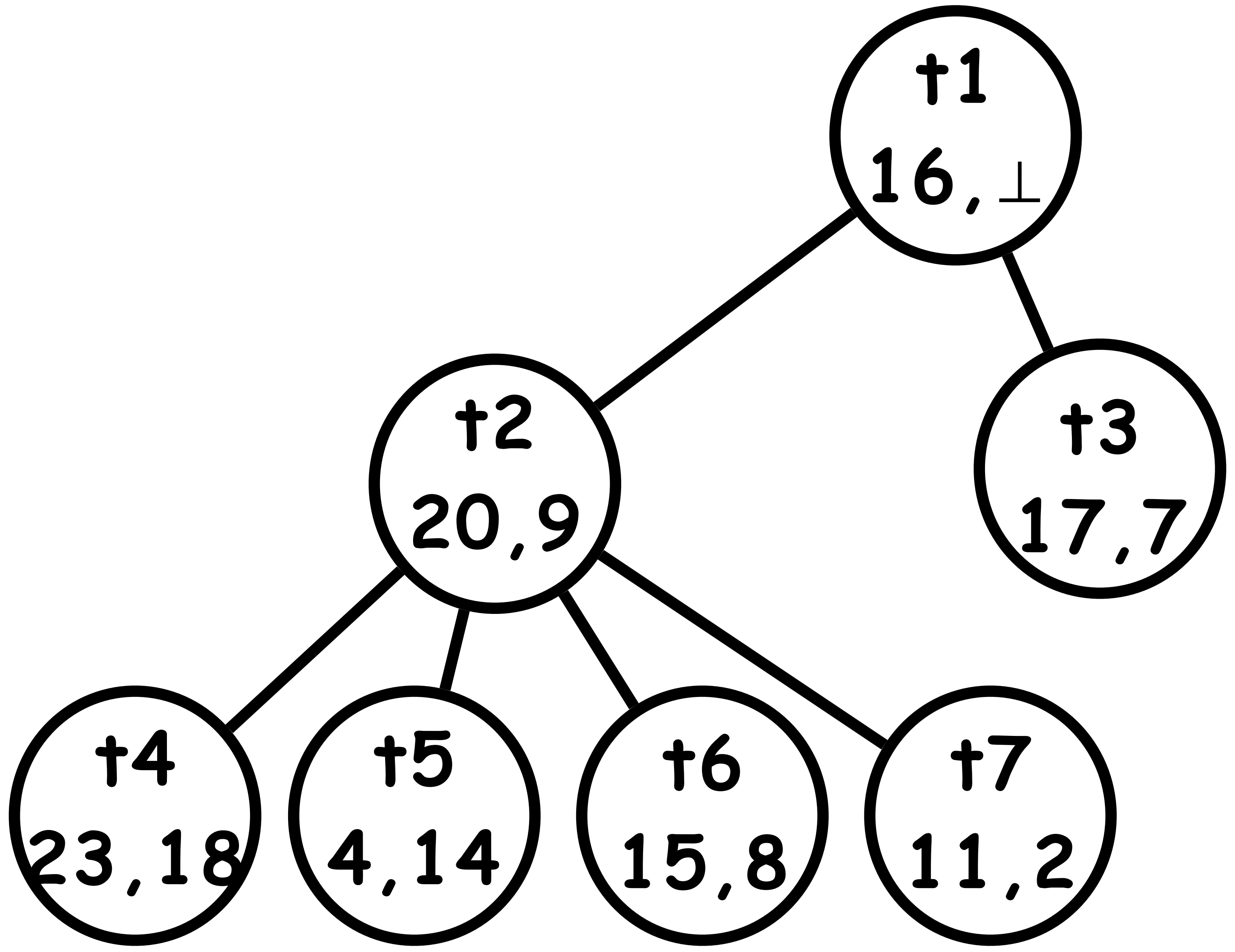


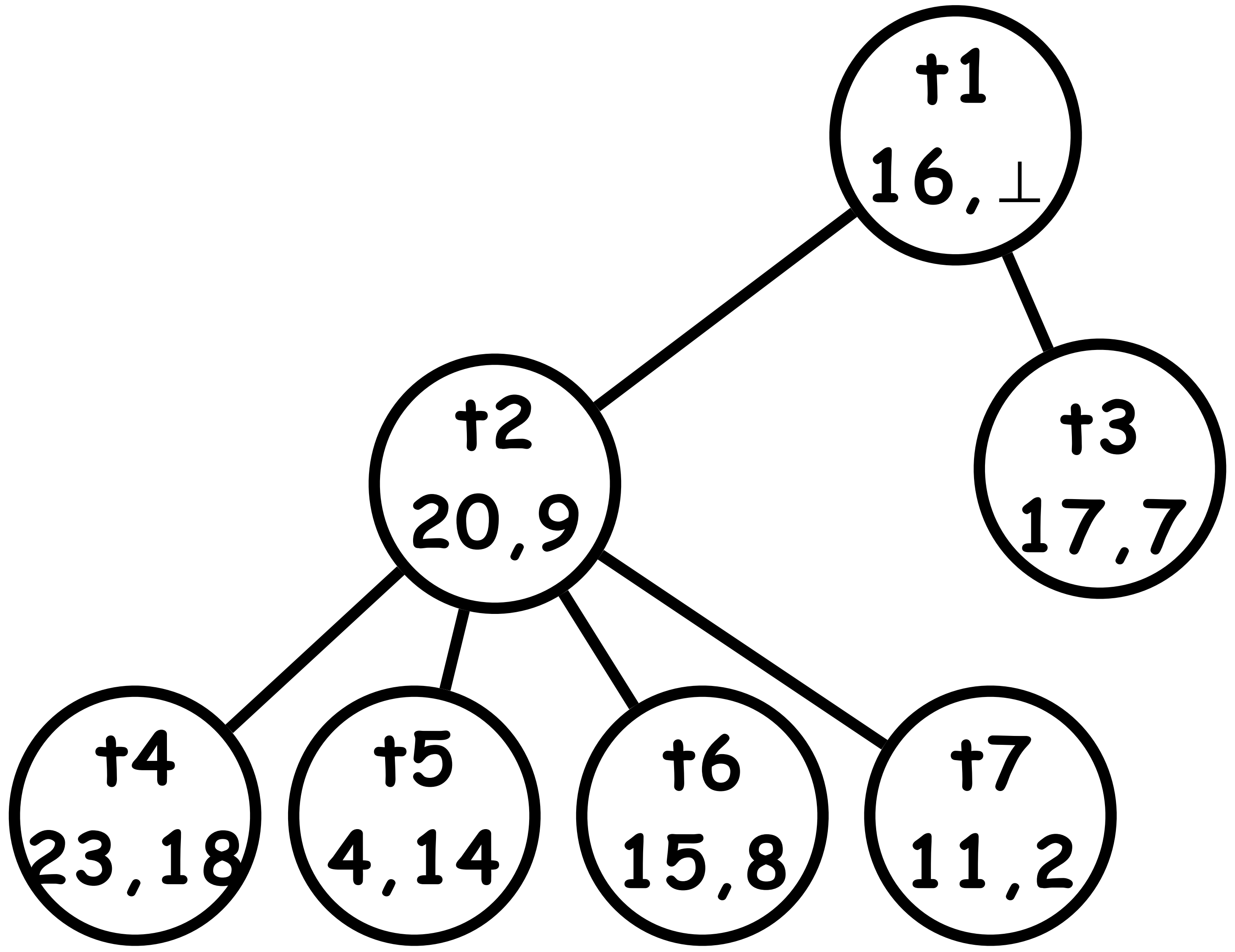
tree clock

Tree Clock

- Implementing logical clocks using generic trees
- **Optimal** asymptotic time complexity in performing logical clock operations

$$(t_1 : 16,\ t_2 : 20,\ t_3 : 17,\ t_4 : 23,\ t_5 : 4,\ t_6 : 15,\ t_7 : 11)$$

vector clock



tree clock

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  - Performing structure changing operations on TC_1 according to the stack top node of TC_2
  - $\Longrightarrow$ Manifesting both challenges

Verifying Tree Clock

- Tree clock is originally implemented in Java
  - Faithfully translated into C
- Its functional model: verified in Coq
- Its imperative join operation: verified using Verified Software Toolchain (VST)

Rooting For Efficiency

Table 2. Evaluation: array-based v. pointer-based tree clocks

1	2	3	4	5	6
Trace len.	num.	Avg. len.	Ptr. TC (s)	Arr. TC (s)	Speedup
(0M, 60M]	35	0.14M	0.22	0.16	1.25×
(60M, 112M]	24	102M	162.27	115.32	1.41×
(112M, 136M]	29	125M	206.57	147.22	1.40×
(136M, 215M]	29	169M	222.36	190.72	1.17×
(215M, 1B]	29	391M	657.23	463.32	1.42×
Total	146	31.41	48.90	36.10	1.35×

- Array-based trees do bring efficiency!

Summary

- Array-based trees: performance-oriented implementation of tree structures
- Challenges: structure changing operations and non-recursive tree traversals
- Strategies: dual views and tree splitting
- Case study: verification of tree clock

Mechanised development

Our paper (this talk)

The tree clock paper

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Thank you! 😃